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Comparison of Two Methods for Calculating Properties of Rotating Neutron Stars*

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Abstract: Two numerical schemes of rotational neutron star based on general relativity, Hartle code and Butterworth and Ipser's code(BI code), are introduced and discussed. The numerical results of the two codes with different equations of state(EOSs) are compared, and the results indicate that most properties of rotational neutron star with different EOSs and different numerical schemes are in accordance with each other.

Key words: rotating neutron star; Hartle code; Butlerworth and Ipser's code; equations of state
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1 Introduction

Nowadays, solving the structure of neutron stars rotating with arbitrary angular velocity has no difficulty of principle, but has difficulties in numerically treatment. Instead of two ordinary differential equations in non-rotating case, one has equivalent of an infinite system of ordinary differential equations in rotating case — one for each coefficient of an expansion of all relevant quantities in spherical harmonics. An exact numerical solution for arbitrary angular velocity seems formidable. So several different approximate solution of this problem were developed[1—4]. In this paper, Hartle code and BI code will be introduced and discussed, and the numerical results of the two codes with different equations of state (EOSs) will be compared.

2 Numerical Scheme of Hartle Code[1,2]

In relativity, the space-time geometry of a rotating star in equilibrium is described by a stationary and axisymmetric metric of the form

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + e^{2\psi}(d\phi - \omega dt)^2 + e^{2\mu}d\theta^2, \quad (1)$$

where $\omega(r)$ is the angular velocity of the local inertial frame and is proportional to the star rotational frequency Ω, which is the angular velocity of the surface of the star relative to an observer at infinity. Accurate to order Ω, from the (t, ϕ) component of Einstein field equations, one gets

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4 j\frac{d\bar{\omega}}{dr}\right) + \frac{4}{r}\frac{dj}{dr}\bar{\omega} = 0, \quad (2)$$

where $\bar{\omega} = \Omega - \omega$, which denotes the angular velocity of the fluid relative to the local inertial frame, $j(r) = e^{-\varphi} \cdot [1 - 2M_0(r)/r]^{1/2}$. The boundary conditions are imposed as $\bar{\omega} = \bar{\omega}_c$, $d\bar{\omega}/dr|_{\bar{\omega}_c} = 0$, where $\bar{\omega}_c$ is chosen arbitrarily. Integrating eq. (2) outward from the center, one can get the function $\bar{\omega}(r)$.

Expanded the metric function through second order in Ω, from the (t,t) and (r,r) components of

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Einstein field equations, one gets two coupled ordinary differential equations of h_0 and m_0 as

$$\frac{dm_0}{dr} = 4\pi r^2 \frac{d(\rho + p)}{dP}(\rho + p)p_0^* + \frac{1}{12}j^2 r^4 \left(\frac{d\bar{\omega}}{dr}\right)^2 - \frac{1}{3}r^3 \frac{d(j^2)}{dr}\bar{\omega}^2, \quad (3)$$

$$\frac{dp_0^*}{dr} = -\frac{m_0(1 + 8\pi r^2 p)}{[r - 2M_0(r)]^2} - \frac{4\pi r^2(\rho + p)}{r - 2M_0(r)}p_0^* + \frac{1}{12}\frac{r^4 j^2}{r - 2M_0(r)}\left(\frac{d\bar{\omega}}{dr}\right)^2 + \frac{1}{3}\frac{d}{dr}\left[\frac{r^3 j^2 \bar{\omega}^2}{r - 2M_0(r)}\right], \quad (4)$$

where $p_0^* = -h_0 + \frac{1}{3}r^2 e^{-2\nu}\bar{\omega}^2 + C$, here C is a constant determined by the demand that h_0 be continuous across the star surface. These equations are also integrated outward, with boundary conditions that both m_0 and p_0^* vanish at the origin.

Using above result, the effection of the rotation could be calculated. With the same central density, the difference between the mass of the rotating star and the non-rotating star is

$$\delta M = m_0(R) + \frac{J^2}{R^3}, \quad (5)$$

where $J = (1/6)R^4 d\bar{\omega}/dr|_{r=R}$, R is the mean radius of the rotating star.

3 Numerical Scheme of BI Code[3]

The BI code solves the four field equations following the Newton-Raphson linearization and iteration procedure. One starts with a non-rotating or slow-rotating model and increases the angular velocity in small steps, treating a new rotating model as a linear perturbation of the previously computed rotating model. In BI code, each linearized field equation is discretized, while in Hartle code the equations of the perturbed metric function are coupled, so the four field equations and the hydrostatic equilibrium equation are solved separately.

In the BI code, the metric of a rotating star in equilibrium is written as

$$ds^2 = -e^{2\nu}dt^2 + e^{2\zeta-2\nu}(dr^2 + r^2 d\theta^2) + r^2\sin^2\theta B^2 e^{-2\nu}(d\phi - \omega dt)^2. \quad (6)$$

The Bardeen and Waginer projected field equations[3] (BW equations) will be used as the Einstein field equation as following

$$\nabla \cdot (B\nabla\nu) = \frac{1}{2}r^2\sin^2\theta B^3 e^{-4\nu}\nabla\omega \cdot \nabla\omega + 4\pi B e^{2\zeta-2\nu}\left[\frac{(\rho + p)(1 + \nu^2)}{1 - \nu^2} + 2p\right], \quad (7)$$

$$\nabla \cdot (r^2\sin^2\theta B^3 e^{-4\nu}\nabla\omega) = -16\pi r\sin\theta B^2 e^{2\zeta-2\nu}\frac{(\rho + p)\nu}{1 - \nu^2}, \quad (8)$$

$$\nabla \cdot (r\sin\theta\nabla B) = 16\pi r\sin\theta B e^{2\zeta-2\nu}p, \quad (9)$$

and ζ, $_\mu$, which is a long equation (please see Ref. 3), where ∇ is the 3-dimensional derivative operator in a flat 3-space with spherical coordinates r, θ, ϕ.

The angular expansions of the metric potentials could be written as

$$\nu = \sum_{l=0}^{\infty}\nu_{2l}(r)P_{2l}(\mu), \quad (10)$$

$$\omega = \sum_{l=0}^{\infty}\omega_{2l}(r)P_{2l+1,\mu}(\mu), \quad (11)$$

$$B = \sum_{l=0}^{\infty}B_{2l}(r)T_{2l}^{\frac{1}{2}}(\mu), \quad (12)$$

where $P_l(\mu)$ is a Legendre polynomial, $T_l^{\frac{1}{l}}(\mu)$ is a Gegenbauer polynomial.

The method for numerical calculating the models is as follows: given the EOS, injection energy β and a set of solutions ν, ω, B, ζ, p for a small Ω, one increases the angular velocity in small steps, treating a new rotating model as a linear perturbation of the previously computed rotating model, as each liearized field equation is discretized, the linear system could be solved.

4 Comparison of the Numerical Result

Using Hartle code, Weber et al[5] investigated the influence of rotation on the bulk properties of neutron star by several "modern" EOS[6—8]. The result are presented in Table 1. They attempt to improve on Hartle code to obtain a more accurate estimate of the angular velocity at the mass-shedding limit. Form their work, we know that Hartle code cannot have a sufficient accuracy to compute models of rapidly rotating relativistic stars.

Friedman et al[9] extended the BI code to obtain a large number of rapidly rotating models based on a variety of realistic EOSs[10—12]. The result are presented in Table 1. From the work of Friedman et al, we know that the Kepler angular velocity Ω_K is substantially small than the value for the spherical model. The ranges of Ω_K are from 55% of its spherical value for softest EOS to 75% of its spherical value for stiffest EOS.

Table 1 Rotating neutron star's property with "modern" EOS*

		EOS	ρ_C /(10^{15} g · cm^{-3})	R_0 /km	M_0 $M_\odot$	Ω_K /10^3 s^{-1}	R_e km	e	δM M_0	I (10^{44} g · cm^{-2})
Hartle	→	HV	1.40	12.6	1.88	9.2	14.8	0.73	0.20	23.0
code	Stiffer	Λ^{00}_{Bonn} + HV	1.40	12.0	1.87	9.8	14.2	0.74	0.20	22.4
		HFV	1.70	11.3	2.14	11.8	13.0	0.73	0.18	24.5
BI	→	G	5.50		1.36	15.2	8.6	0.62	0.14	8.60
code	stiffer	FP	2.50		1.97	12.3	12.0	0.67	0.17	24.1
		L	1.11		2.65	7.6	17.3	0.69	0.20	78.7

* $\Omega_K (= e^{\nu-\psi}\nu + \omega \approx \frac{2}{3}\Omega_c)$ is the Kepler angular velocity, the other notation is the same as in Table 1.

The effect of rotation on the moment of inertia is that the effect is greatest on the stiffest EOS with 170% of its spherical value, while on the softest EOS there is still 60% increase.

To the same value of the total mass M of the star, the central density ρ_c of the non-rotating spherial model is bigger than that of the rotating model with Kepler anguler velocity, while the radius R of the non-rotating model is small than that of the fastest rotating model.

Using Hartle code, Schramm[13] studied the properties of rotating neutron star by a generalized chiral $SU(3)$-flavor model. From his work, we could know that: (1) Compared to Newtonian value of Ω_c, there is a decrease in Kepler angular velocity Ω_K of about 25% due to the general relativity corrections; (2) The mass increase due to the rotation will be up to 30% at the Ω_K, which is bigger than that of all above models, and the increase of the radius will be up to 10% close to Ω_K. The "modern" EOS with chiral $SU(3)$-flavor is more near the reality of the matters in neutron star, the result of this model ought to be more believable. It is exciting that most result of this model is in accordance with the result of other above models.

Comparing these result with the observed result of neutron stars, the softer EOS will be given up, because the rotational periods in the millisecond range (for example, P(PRS 1937 + 21) = 1.6 ms) will be small than the Kepler period of the models with softer EOS.

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两种计算旋转中子星性质方法的比较*

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摘 要: 对两种基于广义相对论的计算旋转中子星性质的数值方法: Hartle 法和 Butterworth 和 Ipser 法(简记为 BI 法)进行了较详细的介绍、讨论和比较. 对基于不同物态方程给出的旋转中子星的性质进行对比, 结果表明两种方法给出的相关结果例如旋转中子星的质量、半径和形变等基本一致, 特别是在观测值范围内的计算等均能较好地解释观测结果.

关键词: 旋转中子星; Hartle 方法; Butterworth-Ipser 方法; 物态方程

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