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Properties of Dense Matter and Equation of State for Supernovae and Neutron Stars

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Abstract: The properties of dense matter are studied within the relativistic mean-field theory. The equation of state (EOS) of dense matter are constructed covering a wide range of temperature, proton fraction, and density for the use of supernova simulations. The relativistic mean-field theory is employed to describe the uniform matter, while the Thomas-Fermi approximation is adopted to describe the non-uniform matter. The EOS of neutron star matter is discussed with the inclusion of hyperons. It is found that the EOS at high density can be significantly softened by the inclusion of hyperons. The $^1S_0$ superfluidity of Λ hyperons may exist in massive neutron stars.

Key words: relativistic mean-field theory; equation of state; supernova; neutron star
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1 Introduction

The study on the properties of dense matter has been a hot topic in recent years[1−2]. The equation of state (EOS) of dense matter plays an important role in various astrophysical phenomena such as supernova explosions and the formation of neutron stars[3−4]. Great efforts have been made to derive the EOS for supernova simulations and neutron stars[5]. Most of the investigations were restricted to the case of zero temperature or high density with uniform distribution of particles, which were mainly aimed at the study of neutron stars, but not applicable for the use in supernova simulations. So far, there exist only two realistic EOS's which are commonly used in supernova simulations, namely the one by Lattimer and Swesty[5] and the one by Shen et al.[6]. The Lattimer-Swesty EOS is based on a compressible liquid-drop model with a Skyrme force. The Shen EOS is based on a relativistic mean-field (RMF) model and uses the Thomas-Fermi approximation in a Wigner-Seitz cell for the description of non-uniform matter. Recently, a Hartree mean-field calculation was performed for the Wigner-Seitz cell in non-uniform matter[7]. The Hartree calculation can incorporate nuclear shell effects, but it requires much more computational resources.

We construct the EOS for the use of supernova simulations and neutron star calculations in the framework of the RMF theory. The RMF theory has been successfully and widely used for the description of nuclear matter and finite nuclei[8−14]. In the RMF approach, baryons interact through the exchange of scalar and vector mesons, and parameters are generally determined by fitting to some nuclear matter properties or ground-state properties of finite nuclei. We study the properties of dense matter with both uniform and non-uniform distributions in the RMF framework adopting the TM1 parameter set, which was determined in

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Ref. [15] by fitting some ground-state properties of nuclei including unstable ones. For uniform matter, the RMF theory can be easily used to calculate the properties of matter. For non-uniform matter where heavy nuclei are formed in order to lower the free energy, we adopt the Thomas-Fermi approximation based on the work by Oyamatsu[16]. The non-uniform matter can be modeled as a mixture of a single species of heavy nuclei, alpha particles, and free nucleons that exist outside of nuclei. The results of the RMF model are taken as input in the Thomas-Fermi calculation, so the treatment of non-uniform matter is consistent with that of uniform matter. We construct the EOS table of supernova matter covering a wide range of temperature T, proton fraction Y_p, and baryon mass density ρ_B for the use of core-collapse supernova simulations[6, 17]. Recently, the Shen EOS table has been updated with an improved design of ranges and grids according to the requirements of the EOS users[18]. In addition, non-nucleonic degrees of freedom, such as hyperons and deconfined quarks, have been considered at high density by several groups[19−21], and their EOS tables are connected with the Shen EOS at low density.

As for the EOS of neutron star matter, the proton fraction is determined by the β-equilibrium condition. At high density, contributions from non-nucleonic degrees of freedom have been extensively discussed in the literature[1−2]. It is generally believed that hyperons appear around twice normal nuclear matter density in neutron star matter[19, 22]. The first hyperon to appear should be Λ that is the lightest hyperon with an attractive potential in nuclear matter. From the experimental binding energies of single-Λ hypernuclei, the potential depth of Λ in nuclear matter is estimated to be $\sim$ 30 MeV[23]. Several recent observations of double-Λ hypernuclei indicate that the effective $\Lambda\Lambda$ interaction should be considerably weaker than that deduced from the earlier measurement[24]. It has been discussed that $^1S_0$ superfluidity of Λ hyperons may exist in neutron stars[25]. Σ hyperons are now considered to appear at a higher density than Λ, because Σ hyperons feel a repulsive potential in nuclear matter according to recent developments in hypernuclear physics[19, 25]. The presence of boson condensation and deconfined quarks in neutron stars has been extensively discussed in many works[1−2, 26−27]. Generally, the introduction of non-nucleonic degrees of freedom leads to a softening of the EOS and reduction in the maximum mass of neutron stars. The recent measurement of the Shapiro delay in the radio pulsar PSR J1614-2230 yielded a mass of $1.97 \pm 0.04\ M_\odot$[28]. Such a high neutron star mass provides an important constraint on the EOS at high density and rules out many predictions of non-nucleonic components in neutron star interiors. However, it is currently difficult to rule out all possible exotica with the $1.97\ M_\odot$ observation[29].

2 Formalism

We briefly describe the RMF theory used in this work. In the RMF theory, baryons interact via the exchange of mesons. The baryons could be nucleons in nuclear matter or including all charge states of the baryon octet in hyperonic matter. The exchanged mesons may include isoscalar scalar and vector mesons (σ and ω), isovector vector meson (ρ), and two additional hidden-strangeness mesons (σ^* and ϕ) that appear only in hyperonic matter. It is well known that the pion plays an important role in many aspects of nuclear physics. However, the pion mean field vanishes in the standard RMF theory, where the effect of the pion exchange is considered to be contained in the model parameters. The contribution of the pion exchange can be taken into account explicitly in some extended RMF models and relativistic Hartree-Fock methods[30−31].

The effective Lagrangian in the mean-field approximation is given as

$$\mathscr{L}_{\rm RMF} = \sum_{B} \bar{\psi}_B [i\gamma_\mu \partial^\mu - (M_B + g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^*) -$$

$$(g_{\omega B}\omega + g_{\phi B}\phi + g_{\rho B}\tau_3\rho)\gamma^0]\psi_B - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{4}c_3\omega^4 + \frac{1}{2}m_\rho^2\rho^2 - \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_\phi^2\phi^2, \quad (1)$$

where ψ_B is the baryon field and the index B runs over the baryon octet (p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-) if all of them are taken into account. This expression can be simplified in several aspects. For matter with nucleons only, the index B denotes p and n, and the hidden-strangeness mesons σ^* and ϕ do not appear. For matter including hyperons, we can neglect the contribution from the exchange of the hidden-strangeness mesons by setting $g_{\sigma^* B} = g_{\phi B} = 0$. It has been found that the attraction from σ^* exchange is almost cancelled by the repulsion from ϕ exchange[23]. If we only include Λ hyperons in addition to nucleons, the index B runs over p, n, and Λ.

We adopt the TM1 parameter set for the nucleonic sector, which can provide a good description of nuclear matter and finite nuclei including unstable ones[15]. As for the meson-hyperon couplings, we take the naive quark model values for the vector coupling constants, while the scalar coupling constants are chosen to give reasonable hyperon potentials[19, 25]. With the effective Lagrangian given in Eq. (1), the derivation of the Euler-Lagrange equations and the following treatment for the EOS of uniform matter can be done directly.

In the low-temperature and low-density region where heavy nuclei are formed in order to lower the free energy, we adopt the Thomas-Fermi approximation based on the work by Oyamatsu[16]. In this approximation, the non-uniform matter at finite temperature is modeled as a mixture of free neutrons, free protons, α-particles, and a single species of heavy nuclei, while the leptons are treated as uniform non-interacting particles separately. For the non-uniform matter at zero temperature, there are no free protons and α-particles outside the heavy nuclei. We assume that each heavy nucleus is located in the center of a charge-neutral cell, and the heavy nuclei form a body-centered-cubic (BCC) lattice to minimize the Coulomb lattice energy. It is useful to introduce the Wigner-Seitz cell to simplify the energy of the unit cell. We assume the nucleon distribution functions $n_i(r)$ (i=n for neutron, i=p for proton) in the Wigner-Seitz cell as

$$n_i(r) = \begin{cases} (n_i^{in} - n_i^{out})\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{out}, & 0 \leqslant r \leqslant R_i \\ n_i^{out}, & R_i \leqslant r \leqslant R_{cell} \end{cases} \quad (2)$$

where r represents the distance from the center of the nucleus, and R_{cell} is the radius of the Wigner-Seitz cell defined by the relation $V_{cell} = 4\pi R_{cell}^3/3 = a^3$ (a is the lattice constant). The density parameters n_i^{in} and n_i^{out} are the densities at $r=0$ and $r \geqslant R_i$, respectively. The parameters R_i and t_i determine the boundary and the relative surface thickness of the nucleus. R_n and t_n may be different from R_p and t_p due to the additional neutrons forming a neutron skin in the surface region. For a system with fixed T, Y_p, and baryon density ρ_B, the optimal state is determined by minimizing the free energy density with respect to the independent parameters in the approximation. The free energy density contributed by baryons is presented by $f=(E-TS)/a^3$, where a is the lattice constant, $E=E_{bulk}+E_S+E_C$ is the energy per cell. The bulk energy E_{bulk} and the entropy per cell S can be calculated by $E_{bulk} = \int_{cell} \varepsilon_{RMF}\, d^3 r$ and $S = \int_{cell} s_{RMF}\, d^3 r$, where ε_{RMF} and s_{RMF} are the energy density and entropy density obtained in the RMF theory. For the surface energy E_S and the Coulomb energy E_C, we use the formula given in Ref. [16]. The system prefers the BCC lattice because it gives the lowest Coulomb energy.

3 Results

We construct the EOS table covering a wide range of temperature T, proton fraction Y_p, and baryon mass density ρ_B for the use of supernova simulations. At each fixed T, Y_p, and ρ_B, the thermodynamically favorable state is determined by performing the minimization of the free energy density. We decide the phase transition from non-uniform matter to uniform matter by comparing their free energy densities. In Fig. 1, we show the phase diagram in the ρ_B-T plane for $Y_p=0.1$, 0.3, and 0.5. The shaded region corresponds to the non-uniform matter phase where heavy nuclei are formed in order to lower the free energy. The dashed line is the boundary where the α-particle fraction X_α changes between $X_\alpha<10^{-4}$ and $X_\alpha>10^{-4}$. We find that heavy nuclei can exist in the medium-density and low-temperature region. At low density, the thermodynamically favorable state is homogeneous nucleon gas with a small fraction

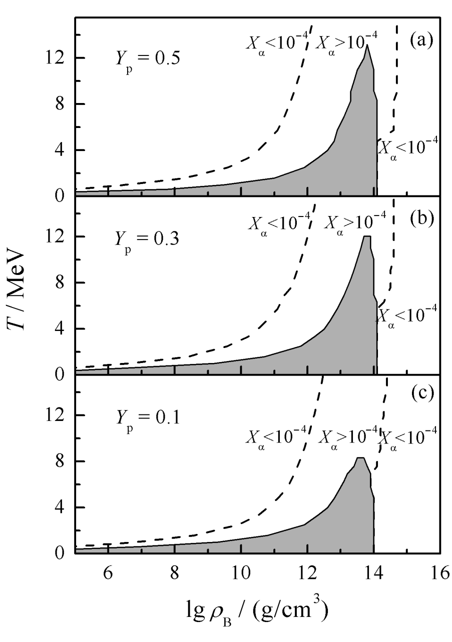

Fig. 1 Phase diagram of dense matter at $Y_p=0.1$, 0.3, and 0.5 in the ρ_B-T plane. The shaded region corresponds to the non-uniform matter phase where heavy nuclei are formed. The dashed line is the boundary where the α-particle fraction X_α changes between $X_\alpha<10^{-4}$ and $X_\alpha>10^{-4}$.

of α-particles. The heavy nuclei are formed at some medium densities where the free energy of the system could be reduced by forming heavy nuclei. The optimal state is a uniform matter when the density increases beyond $\sim 10^{14.2}$ g/cm^3. It is shown that the starting density of the non-uniform matter phase depends on T strongly, while the ending density is nearly independent of T. As the temperature increases, the density range of the non-uniform matter phase becomes narrower, and it disappears completely at higher temperature.

For neutron star matter, we construct the EOS at zero temperature covering a wide density range. The proton fraction in neutron star matter is determined by the β-equilibrium condition, and leptons should be incorporated in order to keep charge neutrality. The properties of the EOS for neutron stars within the nucleon degree of freedom have been discussed in Ref. [21], while the contribution of hyperons has been shown and discussed in Refs. [22, 25]. In Fig. 2, we show the particle fraction, $Y_i=\rho_i/\rho_B$, as a function of the baryon density, ρ_B, using the RMF theory with the TM1 parameter set[15]. It is seen that Λ hyperons appear around 0.31 fm^{-3}, and then increase rapidly with increasing density. Ξ^- and Ξ^0 appear at $\sim$ 0.41 and 0.75 fm^{-3}, respectively. Σ hyperons do not appear at $\rho_B<1.0$ fm^{-3} due to their repulsive potential. We note that hyperon threshold densities, fractions, and effective masses are dependent on the RMF parameters used.

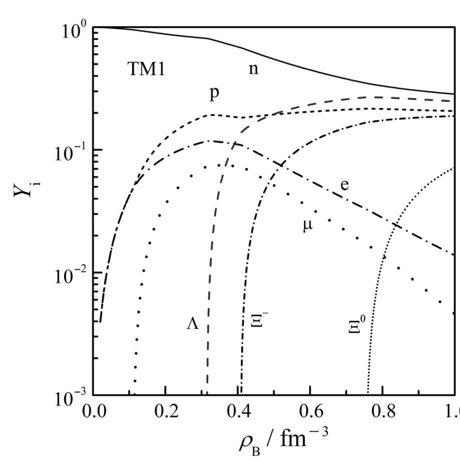

Fig. 2 Particle fraction $Y_i=\rho_i/\rho_B$ as a function of baryon density ρ_B.

We investigate the possibility of appearance of Λ superfluidity in neutron stars[25]. We study the $^1S_0$ superfluidity of Λ hyperons in neutron star matter. With the effective mass and the Fermi momentum of Λ hyperons obtained in the RMF approach, the gap equation can be solved numerically. We adopt several $\Lambda\Lambda$ pairing interaction in the $^1S_0$ channel. Most of them are based on the Nijmegen models and used in double-Λ hypernuclei studies[32-33]. The ND1 potential was given in Ref. [32] as an effective soft-core interaction fitted to the Nijmegen model D (ND) hard-core interaction. Another simulation of the ND interaction, referred to as ND2, was obtained in Ref. [33]. The ESC00, NSC97b, NSC97e, and NSC97f potentials given in Ref. [33] were obtained by changing the strength

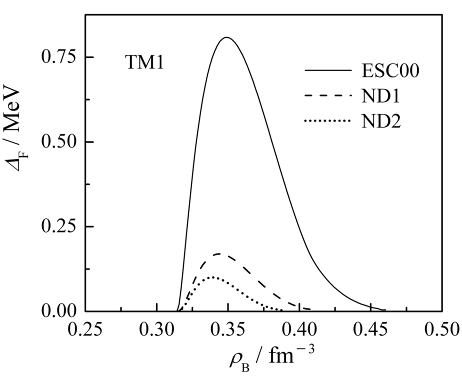

Fig. 3　$^1S_0$ pairing gap of Λ hyperons at the Fermi surface Δ_F as a function of baryon density ρ_B in neutron star matter with the ESC00, ND1, and ND2 potentials.

of the medium-range attractive component of the three-range Gaussian potential, so that they could reproduce the scattering length and the effective range as close to values by the corresponding Nijmegen models. In Fig. 3, we present the $^1S_0$ pairing gap of Λ hyperons at the Fermi surface. It is found that the maximal pairing gap is about 0.8 MeV with the ESC00 potential. This is because the ESC00 potential has the strongest attraction among the $\Lambda\Lambda$ interactions used. The pairing gaps with the ND1 and ND2 potentials are of the order of 0.1～0.2 MeV. For other potentials, the pairing gaps are either very small values or absent. This reflects the dependence of pairing gaps on the $\Lambda\Lambda$ interaction. We examine whether the $^1S_0$ superfluidity of Λ hyperons can exist in neutron stars. By solving the Tolman-Oppenheimer-Volkoff equation with the EOS obtained in the RMF theory, we find that whether the $^1S_0$ superfluidity of Λ hyperons exists in the core of neutron stars depends on the $\Lambda\Lambda$ interaction used. With stronger $\Lambda\Lambda$ interactions, such as ESC00, ND1, and ND2, the $^1S_0$ superfluidity of Λ hyperons may exist in massive neutron stars.

4 Summary

We have studied the properties of dense matter within the RMF theory. The EOS are constructed covering a wide range of temperature, proton fraction, and density for the use of supernova simulations. The RMF theory is employed to describe the uniform matter, while the Thomas-Fermi approximation is adopted to describe the non-uniform matter. The properties of dense matter with both uniform and non-uniform distributions are studied in a consistent manner. We have discussed the EOS including hyperons and the possibility of appearance of Λ superfluidity in neutron stars. It is found that the EOS at high density can be significantly softened by the inclusion of hyperons. With stronger $\Lambda\Lambda$ pairing interaction, the $^1S_0$ superfluidity of Λ hyperons may exist in massive neutron stars.

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致密物质性质和适用于超新星及中子星研究的状态方程

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摘要: 采用相对论平均场方法研究了致密物质的性质，构造了包括较宽温度、同位旋不对称度和密度范围的适用于超新星模拟研究的状态方程，均匀物质由相对论平均场理论描述，非均匀物质由托马斯-费米近似给出。讨论了包含超子自由度的中子星物质的状态方程。计算结果表明，包含超子可以有效地软化高密度区的状态方程，Λ 超子的超流态有可能存在于大质量中子星内部。

关键词: 相对论平均场理论; 状态方程; 超新星; 中子星

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