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Theoretical Studies of Proton Radioactivity

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Abstract: In the paper, we will discuss the most recent theoretical approaches developed by our group, to understand the mechanisms of decay by one proton emission, and the structure and shape of exotic nuclei at the limits of stability.

Key words: proton radioactivity; drip-line; nuclear density functional theory

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1 Introduction

Proton radioactivity has provided a unique probe to study the structure of nuclei at the proton drip line. The small production cross section, and the very short life times involved in the study of these nuclei, motivated a considerable improvement in detection and production techniques in the last decade, and a large number of one and two proton emitters have been observed. In some cases, using tagging techniques, the spectrum was also mapped, and the γs from the electromagnetic transitions detected. Many more proton emitters are still expected to exist, so the production of new exotic beams, and the capacity to observe half-lives below a μs, will be a challenge for the future.

In this context, the need to have a solid theoretical ground, with great independence from parameterizations, and with the capability to interpret the data and help the search of new emitters, is crucial.

It is the purpose of this contribution to discuss the current stage of the theoretical approaches developed by our group, within a non-relativistic and a relativistic covariant density functional formalisms, to describe proton radioactivity.

Within the non-adiabatic quasi-particle model, we will address the example of ground state and isomeric decays of ^{151}Lu, and study decay of ^{131}Eu in a fully self consistent calculation, with interactions derived from covariant density functional theory.

2 Recent results

2.1 The deformation of ^{151}Lu

Since the 1980's various experiments were devoted to the observation of ^{151}Lu[1], and the knowledge gathered, reflects the evolution on the production and detection of nuclei at the extremes of stability. Whereas in the first experiment[1], only proton emission from the ground state was observed, decay from isomeric states, expected in similarity with what is observed in neighbouring nuclei, was not seen, since they involved very small production cross sections and also half-lives of few microseconds, unobservable in those days. In a later experiment[2] proton emission was also observed from a low-lying isomeric state with a very small half-life of 16(1) μs.

Recently, a new experiment[3–4] was devoted to provide better understanding of the decay of this nucleus and to establish the value of its deformation, which relies primarily on the knowledge of the low-lying excited states already discussed in the work of Refs. [5-6]. Using a combination of recoil decay tagging and recoil-distance Doppler-shift techniques, the level structure of ^{151}Lu above the proton emitting states, and the electromagnetic lifetime of the first excited state, as well as higher-lying longer-lived states were measured. An updated value for the energy of the outgoing proton during the decay of the isomer was also determined, *i.e.* $E_{\rm p} = 1\,285(4)$ keV updating the older value $E = 1\,310(10)$ keV[2].

The theoretical interpretation of the data was initially based on the assumption that the nucleus was spherical, being close to the $N = 82$ shell closure, and used a spherical WKB barrier penetration analysis[2]. However, whereas the ground state decay was interpreted as decay from the $h_{11/2}$ single particle state, the isomeric decay, attributed to decay from the $d_{3/2}$

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state, required a larger BCS spectroscopic factor than the so-called experimental one, defined as the ratio between the theoretical and the measured half-life[2]. This was interpreted as a possible indication of the coupling of core excitations to phonon states. However, models based on the mass formula[7] or relativistic Hartree-Bogoliubov calculations[8], predicted for ^{151}Lu a reasonably large deformation. The possibility of deformation was analyzed in Ref. [9] within the adiabatic approach, where the emitter is described as a rotor of infinite momentum of inertia plus one particle. The spectroscopic factor needed for a comparison with the experiment, was obtained from a BCS calculation, and measures the probability that the state is empty in the daughter nucleus. The comparison with the experimental data indicated that a deformation $-0.18 < \beta_2 < -0.14$, could interpret decay from a $5/2^-$ ground state and from a $3/2^+$ isomeric state. The deformation needed in this interpretation of data is very close to the ones predicted in Refs. [7-8].

The most consistent non-relativistic theoretical approach to describe proton emission from deformed nuclei, is the non-adiabatic quasiparticle approach[10], which has been applied very successfully to odd-even and odd-odd emitters[11], and can also account for the breaking of axial symmetry[12]. Therefore, the most recent data[3–4] was interpreted within this model.

Within the model, the proton is in a single particle Nilsson resonance with the deformed core, and the excitation spectrum of the daughter nucleus is taken into account. The total Hamiltonian corresponding to the system can be written as,

$$H = H_{\rm int} + H_{\rm rot} \ , \tag{1}$$

where $H_{\rm int}$ includes the Nilsson Hamiltonian and the pairing residual interaction, and the rotational hamiltonian $H_{\rm rot}$, includes the component corresponding to pure rotations and the Coriolis interaction, if the nucleus is a perfect rotor, or otherwise, the experimental spectrum of the core, and the coupling of the proton to the core. According to the non-adiabatic quasi-particle model[10], to take correctly into account the pairing interaction, $H_{\rm rot}$ has to be diagonalized between quasiparticle states. The half-life for decay by one particle emission can be determined from general scattering theory, as described in Ref. [10].

Since the spectrum of the daughter nucleus ^{150}Yb is not known, we have taken from Ref. [13] the spectrum of ^{148}Er, which is a close neighbour, and should have essentially the same deformation. The yrast band of ^{148}Er shows that it is not a pure rotor.

We have diagonalized the interaction of Eq. 1, and calculated the energy levels of ^{151}Lu. The spectrum of the negative parity states built on the ground state, are shown in Fig. 1 as a function of deformation. For deformations $\beta_2 > -0.3$ the $h^-_{11/2}$ level is predicted to be the ground state. The half life for decay from this state calculated within our model is also compared with the experimental one in the same figure. The error in the theoretical prediction is due to the uncertainty in the knowledge of the experimental energy of the outgoing proton, and on a possible shift up or down of the Fermi energy. The relative position of the single particle levels is quite sensitive to details of the nuclear interaction. Changing the relative position between positive and negative parity states, that depends strongly on the spin-orbit interaction, will change the Fermi energy, and so also the occupation probability of the states, the spectroscopic factor, and consequently the half-life.

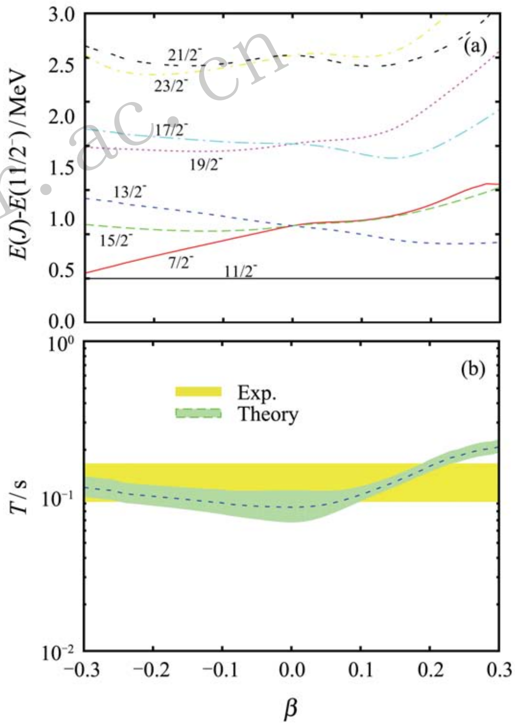

Fig. 1　(color online) (a) theoretical negative parity states in ^{151}Lu, within the non-adiabatic quasi particle model, as a function of deformation. (b) theoretical half-life for decay from the $h_{11/2}$ state as a function of deformation (green line), in comparison with the experimental value (yellow line).

The comparison between the theoretical and experimental half-lives excludes a prolate deformation large than $\beta = 0.2$. Since the experimental spectrum shows that the $13/2^-$ lies above the $15/2^-$, and the $17/2^-$ above the $19/2^-$, this is only possible if ^{151}Lu has a mild oblate deformation, as it can be understood from the observation of Fig. 1(a), where the theoretical levels are displayed.

The half-life for the electromagnetic decay of the $15/2^-$ state is shown in Fig. 2 , as a function of deformation. obtained within the non-adiabatic quasi-particle description of the nucleus. A comparison with the experimental value of 7.4(42) ps, allows to define a region of deformation, where theory and experiment are compatible. As it can be seen from Fig. 2, to reproduce the experimental half-life for γ decay, the deformation should be in the range of $-0.09 > \beta_2 > -0.16$.

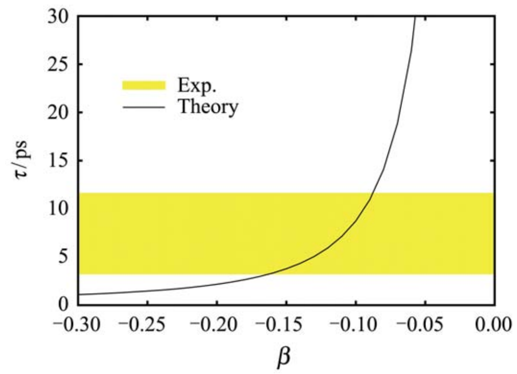

Fig. 2 (color online) Half-life for the electromagnetic decay of the $15/2^-$ state in ^{151}Lu, as a function of deformation. The yellow band, defines the experimental value of 7.4(42) ps. The intersections of the theoretical line with the experimental region, define the possible range of deformation.

This is a very interesting result, since it shows the consistency of the theoretical calculation in its capacity to reproduce experimental results from very different origin, *i.e.*, life times for proton emission and gamma decay, and the excitation spectrum.

The deformation for all the other states, can be obtained in an analogous manner, showing an increase in quadrupole deformation with increasing spin, as shown in Table 1, due to the increased influence of high angular momentum configurations in the Nilsson states at the Fermi surface. For details see Ref. [4].

Table 1 Level scheme above the isomeric state of ^{151}Lu, and corresponding quadrupole deformation.

J^π	β_2	J^π	β_2
$23/2^+$	−0.18	$11/2^+$	−0.12
$19/2^+$	−0.18	$7/2^+$	−0.11
$15/2^+$	−0.15	$3/2^+$	−0.11

Decay from the isomeric state can be interpreted within the same model. The levels $1/2^+$ and $3/2^+$, for some values of the deformation can lie quite closely in energy, and even have their order reversed. But the comparison with the experimental data shows that the theoretical half-life is only consistent with proton emission from a $3/2^+$ isomeric state, as it was already suggested in Ref. [2], and which has a quadrupole deformation of $\beta_2 = -0.11$. In Fig. 3, the half lives, were also calculated considering the proton was at the old[2] experimental energy $E_\mathrm{p} = 1310(10)$ keV, but the consistency with the previous results for an oblate quadrupole deformation of ^{151}Lu, supports the most recent measurement for the energy.

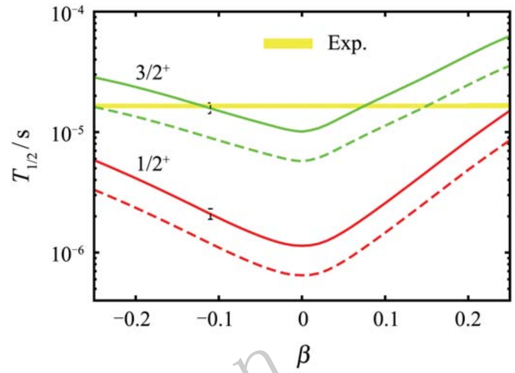

Fig. 3 (color online) Half-life for proton emission from various states as a function of deformation, for the new value of the proton energy (full lines) and with the old data (dashed lines) compared with the experimental result.

From the above discussion, we can see that the knowledge of the deformation and of the low-lying excited states in ^{151}Lu, and their lifetimes is now well established. Theoretical calculations using the non-adiabatic quasi-particle approach,suggest that the $h_{11/2}$ state is the proton-emitting ground state, with a mildly oblate deformation of $\beta_2 = (-0.11^{+0.02}_{-0.05})$. Decay from the isomeric state was identified with decay from the $3/2^+$ with a quadrupole deformation of $\beta_2 = -0.12(1)$.

2.2 Covariant density function theory for decay of deformed proton emitters

Nuclear density functionals have been very successful in describing ground-state properties and collective excitations of nuclei, from very light systems up to super-heavy elements, and from the stability region up to exotic nuclei at the limits of stability[14–15]. Ground-state quadrupole deformations have been predicted for proton rich nuclei, and proton separation energies have been calculated, allowing the proton drip-line to be sketched.

The majority of nuclear structure calculations, rely on the various parameterizations of the nuclear mean field available in the literature, which are obtained from fits to the single particle properties of nuclei or by semiclassical considerations based on the liquid drop model and the proximity force. There is no interactions yet from first principles.

Covariant density functional theory (CDFT), has many advantages. As a relativistic quantum field approach, it has a density dependence to allow for a quantitative description of nuclear surface properties, and the treatment of the spin-orbit interaction arises in a natural way, without any additional adjustable parameters. The empirical pseudospin symmetry is also explained, and the model is also consistent with the non linear realization of chiral symmetry. Therefore, CDFT needs only a relatively small number of parameters determined by a global fit to ground state properties of nuclei and to nuclear matter properties. The functionals can be considered universal since they are valid all over the periodic table, where mean field theory is applicable.

In this theory, the nucleons are described by the Dirac spinors ψ, interacting in an effective Lagrangian through the exchange of the isoscalar scalar σ, isoscalar vector ω, and isovector vector ρ mesons, and the electromagnetic field. This simple model, with interaction terms linear in the meson fields, does, however, not provide a quantitative description of the nuclear surface properties. An effective density dependence can be introduced either by using non-linear coupling terms or by considering coupling constants dependent on the density of the exchanged mesons. Examples of these interactions are the non-linear meson exchange model NL3[16] and the density dependent point coupling models DD-PC1[17], and PC-PK1[18].

From the Lagrangian density the classical variation principle leads to the equations of motion, which are the Dirac equation for the nucleons, equivalent to the Kohn-Sham equations in non-relativistic density functional theory, and the Klein-Gordon equations for the meson fields. Pairing correlations are taken into account in the constant gap approximation. This set of equations is non-linear and is solved by iteration starting with an initial guess for the potentials, until self-consistency is achieved, and the final mean field interaction obtained.

In the past[19], we have performed a fully self-consistent relativistic mean field calculations (RMF) for spherical nuclei, starting with the functionals NL3 and DD-PC1 to obtain the interactions as described previously, and determine the proton resonances and their probability to decay by one proton emission. The calculation was very successful in reproducing the experimental data, and provided a clear evidence for a mixing of configurations.

We have generalized our previous calculation in order to describe proton radioactivity from deformed nuclei[20]. Two different relativistic point-coupling models were used, the DD-PC1 and the PC-PK1 of the Peking group, both successful in describing nuclear properties, but with some differences between them[21–22], regarding their structure and fitting procedure.

The functional PC-PK1 has phenomenological parameters fitted to the binding energies and radii of a large number of spherical semi-magic nuclei all over the periodic table. The functional DD-PC1, was instead fitted to masses of 64 heavy deformed nuclei and a few nuclear matter properties. Therefore, both interactions describe spherical and deformed nuclei differently, being the masses of deformed nuclei better reproduced by the DD-PC1, whereas for spherical nuclei, the masses are over-bound.

The scalar and vector potentials that describe the deformed proton emitter, were obtained in a fully self-consistent way, by solving the relativistic mean field equations (RMF) equations for the even-even daughter nucleus including pairing correlations in the BCS approximation. The resulting self-consistent axial potentials $S(\boldsymbol{r})$ and $V(\boldsymbol{r})$ were then expanded in spherical harmonics with different L-values, obtaining the corresponding potentials $S_L(r)$ and $V_L(r)$ on a fine mesh in r-space. The latter interactions, were then used to calculate the proton Nilsson resonances, which are the solutions of the Dirac equation imposing outgoing wave boundary conditions. This was achieved by solving a full coupled channel problem in coordinate space, leading to a precise description of the energy, width and wave function of the proton resonances.

This is a complex problem, solved previously only for $K=1/2^+$ resonant states by the Beijing group[23], and in Ref. [24] using an extension of the complex scaling method. We have solved the full relativistic coupled channel problem for any value of the angular momentum in coordinate space[20], using similar techniques as in the non-relativistic case[25].

As an example to apply our model, we have chosen the proton emitter ^{131}Eu, a highly deformed nucleus where the radioactive decay from the ground state to the ground and first excited 2^+ states of the daughter nucleus ^{130}Sm, were identified[26–27]. If there is no change of deformation during the decay, ^{131}Eu should have a similar deformation to the one expected for ^{130}Sm, *i.e.* a value $\beta_2 \approx 0.34$, deduced from the empirical relation of Ref. [28].

We have calculated the half-life for decay from the various states close to the Fermi surface and the branching ratios for decay these states to the first excited 2^+ state of the daughter ^{130}Sm. For details, see Ref. [20].

The half-life depends strongly on the quantum numbers of the decaying state, and also on details of the interaction like for example the nuclear radius, and strength of the spin-orbit force. In contrast, the branching ratios are quite stable regarding these quantities, but are very sensitive instead to details of the wave function components with different angular momentum, since their calculation involves the sum over all allowed partial decay widths for decay, divided by the total width, so they are a ratio between very different amplitudes. Therefore, the branching ratio depends on a balance between details of the wave function components, and for this reason it is an excellent probe of nuclear structure properties.

We present in Table 2 the calculation of the branching ratio for decay from the Nilsson single particle states of ^{131}Eu, close to the Fermi energy, to the 2^+ state of the daughter nucleus, assuming they have a quadrupole deformation predicted by the Grodzins relation[28], $\beta_2 = 0.34$. As it can be seen, only the $3/2^+[411]$ state interprets perfectly the experimental value of 0.24(5)[26], in agreement with what was found in the non-relativistic calculation[29], and also the predictions based on microscopic-macroscopic mass calculations[7]. The calculation was done with the DD-PC1 model, but similar conclusions can be drawn if the PC-PK1 is used, since only minor differences in the behaviour of the half-lives was found.

Table 2 Branching ratio for decay from different J^π states of ^{131}Eu to the first excited 2^+ state of the daughter ^{130}Sm, with a deformation $\beta_2 = 0.34$, and for the DD-PC1 model. The experimental branching ratio is 0.24(5)[26].

J^π	Branching	J^π	Branching
$3/2^+$ [411]	0.191	$5/2^+$[402]	0.023
$1/2^+$[411]	0.024	$7/2^+$[404]	0.029
$5/2^+$[413]	0.031		

This is the first fully self-consistent calculation of proton emission from deformed nuclei, with interactions derived from covariant density functionals. It was very successful to reproduced the experimental data, and has provided a new test of the relativistic point-coupling functionals, the DD-PC1 and the PC-PK1.

3 Conclusions

In conclusion, theoretical calculations within the non-adiabatic quasi-particle approach, for decay by proton emission, can interpret the experimental data and provide very precise wave functions that can be also used to interpret the electromagnetic decay of the excited states. The very good agreement with data shown in the example of ^{151}Lu, not a perfect rotor, so the experimental spectrum of the core was taken into account, has shown the consistency of the model, and allowed to identify the decaying state in the cases of ground state and isomeric decay, and the nuclear deformation.

Relativistic mean field models present some advantages with respect to the non-relativistic ones, concerning the parameterization of the nuclear interaction, but a unique parameterization for the Lagrangian, which is able to describe properties of nuclei from light to very heavy, and from the proton drip-line to the neutron one, is still not available. Therefore, it is desirable to provide new test of interaction derived from the relativistic density functionals and observe their performance. Proton radioactivity provides this possibility. We have tested two density dependent point coupling models, the DD-PC1[17] and the PC-PK1[18], and concluded that they could also describe proton radioactive nuclei, and predict their properties.

From the above discussion, one can conclude that a very solid theory exists that can describe proton radioactivity, interpret the data, and predict features of the structure of nuclei at the extremes of stability.

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