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# Several Controversial Issues of $QED_3$

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**Abstract:** Quantum electrodynamics in (2+1) dimensions ($QED_3$) is an important nonperturbative system. This seems relatively simple Abel system, there are several issues that need to be clarified: whether or not the partition function of the system depends on chemical potential; whether or not there exists dynamical chiral symmetric breaking; whether or not the boson can acquires nonzero mass. In this paper, we give an in sight of the traits of $QED_3$ from the dependence of density, temperature and massive boson to discuss those problems.

**Key words:** $QED_3$; dependence of density; massive boson

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## 1 Introduction

$QED_3$ has been widely studied for many years. It has many features similar to quantum chromodynamics (QCD), such as the dynamical chiral symmetry breaking (DCSB) in the massless fermion limit and confinement[1–5]. Moreover, $QED_3$ is superrenormalizable, so it does not suffer from the ultraviolet divergence which is present in $QED_4$. Therefore it can serve as a toy model of more realistic theories such as QCD. Besides, $QED_3$ has been applied to study some problems in condensed matter physics. Especially, $QED_3$ can be regarded as a model for high-$T_c$ superconductivity and fractional quantum Hall effect [6–10].

This Abelian system has many strange features, but it only exists in 2-dimensional space which can be treated as 3-dimension with the vanishing 3rd-dimension. As a typical system of quantum field theory, we need to talk about its basic properties. Whether or not it still has the same properties as the three-dimensional space system. These common issues include: the density effect of the system, the symmetry breaking at finite temperature and boson mass at zero temperature and zero density in this system. In order to further understand the nature of the system, we will proceed from these basic parameters, discuss the basic properties of the anomalous field theory.

## 2 Whether or not $QED_3$ depends on μ?

In Euclidean space, the Lagrangian of $QED_3$ with finite chemical potential μ reads

$$\mathcal{L}=\bar{\psi}(\partial\!\!\!/+\mathrm{i}e A\!\!\!/-\mu\gamma_3+m)\psi+\frac{1}{4}F_{\sigma\nu}^2 , \tag{1}$$

where the four-component spinors are employed and the term $\mu\bar{\psi}\gamma_3\psi(=\mu\psi^\dagger\psi)$ gives the effect of chemical potential. In the absence of the mass term $m\bar{\psi}\psi$, $\mathcal{L}$ has chiral symmetry and the symmetry group is $U(2)$. The original $U(2)$ symmetry reduces to $U(1)\times U(1)$ when the massless fermion acquires a nonzero mass due to nonperturbative effects. Here, we note that the dependence of chemical potential on $QED_3$ is different from that in (3+1)-dimensional QED ($QED_4$). In the case of $QED_4$, the partition function is independent of chemical potential[11, 12]. First, let us explain physically why the partition function does not depend on μ. In $QED_4$, the interaction potential between two charges is the long-range Coulomb potential:

$$V_4(r)=\frac{1}{2}\sum_{i,j}\frac{e_ie_j}{r},$$

where $r=|x_i-x_j|$. The free charge carriers interact repulsively with the long-range Coulomb force, and hence all the net charge resides on the surface. So the partition function does not depend on μ. Now, let us see what happens in $QED_3$. In $QED_3$ the classical interaction potential associated with the dressed photon propagator can be written as:

$$V_3(r)=\frac{1}{2}\sum_{i,j}\frac{e_ie_j}{2\pi[1+\Pi(0)]}\ln(|e_ie_jr|)+h(r)+\mathrm{const} , \tag{2}$$

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where $\Pi(0)$ is the nonnegative but finite infrared value of boson polarization and $h(r)$ falls off at least as fast as $1/r$ [13, 14]. From the potential of $QED_4$, we see that the interaction between electrons in $QED_4$ reveals screening behavior, while the potential of $QED_3$ shows that the interaction between two fermions reveals typical anti-screening features (similar to the color anti-screening in QCD). Since the charge carriers interact attractively with a long-rang force, the net charges cannot reside on the surface, which is quite different from the case of $QED_4$. In other words, the homogeneous "bulk" of the medium cannot be neutral. Because a non-zero chemical potential corresponds to the system which is charged, the partition function in $QED_3$ depends on μ and hence the propagators, phase transition in $QED_3$ are also affected by chemical potential.

## 3 Whether or not we can study chiral phase transition in thermal $QED_3$?

Here, one may accept the absence of true symmetry breaking in 2-dimensional space system at $T > 0$ for a consequence of infrared divergent fluctuations, mandated by the Coleman-Mermin-Wagenr theorem. Of course, the theorem give us a forceful restriction, *i.e.*, nonexistent DCSB in 2-d space including (2+1)-dimensional QCD, and thus there not exists rigorous chiral phase transition which is well-defined in (3+1)-dimensional system.

However, the DCSB in $QED_3$ at finite temperature is studied by a self-consistent calculation of DSE in rainbow approximation in $QED_3$, which likes the mean field theory in condensed matter. In such a theory, the correlation between fluctuations of the order parameter is ignored. This is known to be unreliable for continuous symmetry breaking, since the Coleman-Mermin-Wagner theorem states that there is no continuous symmetry breaking in two-dimensional space. Nevertheless, the mean field transition temperature provides a correct energy scale below which the amplitude of the order parameter becomes finite and its spatial correlation becomes strong and rather long-ranged. In this sense, the mean field transition marks a crossover in the thermodynamic properties. In particular, for a $U(1)$ or $O(2)$ symmetry to be broken, there is in fact an algebraic order below the so-called Kosterlitz-Thouless (KT) transition temperature (the transition can be found in several 2-D systems in condensed matter physics that are approximated by the XY model, including Josephson junction arrays and thin disordered superconducting granular films. More recently, the term has been applied by the 2-D superconductor insulator transition community to the pinning of Cooper pairs in the insulating regime, due to similarities with the original vortex KT transition), a temperature not far from the mean field one. Moreover, in a realistic layered system, the inter-layer coupling can easily drive the system into a true ordered state once the in-plane correlations are already strong, *e.g.*, below the mean field transition temperature.

## 4 Whether or not the boson has nonzero mass in $QED_3$?

(3+1)-dimensional QED, the massless boson (or the photon) gives the lone-range interaction, but reduce a short one at finite temperature and density where photon acquires a nonzero mass. At zero temperature and density, the mass of boson in $QED_3$ will be different from that in $QED_4$. Because of the nonperturbative features of $QED_3$, the gauge boson will acquire a mass through Andson-Higgs mechanism which happens when the gauge field interacts with some scalar filed in the phase with spontaneous gauge symmetry breaking. In physics, this phenomenon occurs in the state of plane superconductivity where spontaneous gauge symmetry breaking appears. In principle, to make sure the gauge field obtaining a mass, we can introduce the additional interaction term between gauge field $A_\mu$ and complex scalar boson field $\phi$:

$$\mathcal{L}' = \mathcal{L} + \mathcal{L}_B \tag{3}$$

$$\text{with} \quad \mathcal{L}_B = [|(\partial_\mu + \mathrm{i}eA_\mu)\phi|^2 - \mu^2|\phi|^2 - \lambda|\phi|^4] \,. \tag{4}$$

$\mathcal{L}_B$ is so-called Abelian Higgs model or relative Ginzburg-Landau model[15] . The scalar field $\phi$ represents the bosonic holons based on the spin-charge separation picture. When $\mu^2 > 0$, the system stays in the normal state and the vacuum expectation value of boson field $\langle\phi\rangle = 0$, so the Lagrangian respects the local gauge symmetry. When $\mu^2 < 0$, the system enters the superconducting state and the boson field develops a finite expectation value $\langle\phi\rangle \neq 0$, then the local gauge symmetry is spontaneously broken and the gauge field acquires a finite mass after absorbing the massless Goldstone boson. The finite gauge field mass is able to characterize the achievement of superconductivity. On the other hand, the gauge field obtains a mass via Anderson-Higgs mechanism implies that the gauge field is in confinement phase, which deduce that the spions and holons are confined in superconducting phase[16]. It is well known that neither spinon nor holon can be observed in high-$T_c$ superconducting experiments, however, a well defined quasi-particle can be observed due to the spin-charge recombination in superconducting phase.

## 5 Conclusions

The phase transition of $QED_3$ has been studied more than 30 years. As a relatively "simple" system, there are three fundamental issues that need to be further studied. Firstly, we analyze its interaction potential of gauge boson and show that the partition function of $QED_3$ should depend on density. Because DCSB occurs in low energy region, we then illustrated that the system can exist DCSB at not high temperature without violating the basic thermodynamic laws. In the last section, we show that the boson may acquire a nonzero mass via Higgs mechanism. Of course, in order to further confirm those observations, we need to study those issues in future.

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# 三维 QED 中的几个重要问题

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**摘要：** 三维量子电动力学是一种看上去比较简单的 Abel 类型的非微扰系统，其本身却有很多需要澄清的基本问题。从该系统是否具有密度依赖性，有限温下是否具有动力学自发对称破却以及规范玻色子可否具有质量这三方面出发，阐述了对三维量子电动力学一些基本问题的看法。

**关键词：** 三维 QED；密度的依赖性；具有质量的玻色子

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