Article ID: 1007-4627(2017) 01-0057-05

Decay Out of Superdeformed Bands in ^{190,192}Pb

GU Jianzhong[1], PENG Bangbao[2], LI Zhenyu[3], YU Guoliang[4]

(*1. China Institute of Atomic Energy, Beijing 102413, China;*
2. Key Discipline Laboratory of Nuclear Safety and Simulation Technology, Harbin Engineering University, Harbin 150001, China;
3. School of Physics and Electronic Science, Guizhou Normal College, Guiyang 550018, China;
4. Department of Mathematics and Physics, North China Electric Power University, Baoding 071003, Hebei, China)

Abstract: The combined method is applied to calculate the angular momentum projected potential energy surfaces (AMPPES) of ^{190,192}Pb. The Supper-deformed(SD) rotational bands of the two nuclei are studied with the AMPPES computed with the Gogny D1S and Skyrme SkP and SLy4 interactions. It is found that there is no pronounced SD band in ^{190}Pb in the case of the Gogny interaction, which is consistent with the experimental observation. A well developed SD band with the Gogny interaction is found in ^{192}Pb. The tunneling width of ^{192}Pb is comparable to that given by the GW approach (Nucl. Phys. A 660 (1999)197) and orders of magnitude larger than that given by the SB approach (Phys. Rev. C 60 (1999) 051305). The influence of the angular momentum projection on the potential energy surfaces is examined in the case of the Gogny interaction for ^{190,192}Pb. It is shown that the angular momentum projection suppresses the barrier separating the SD and ND rotational bands. Higher barriers of the AMPPESs for the two nuclei computed with the Skyrme SkP and SLy4 interactions are obtained compared with those given by the Gogny force. The tunneling width of ^{192}Pb is also big for the Skyrme interactions. We put the SB approach into question which gives only an extremely small spreading width.
Key words: Combined method; decay out of superdeformed bands; angular momentum projected potential energy surfaces

CLC number: O571.6 **Document code:** A **DOI:** 10.11804/NuclPhysRev.34.01.057

1 Introduction

The study of super-deformed (SD) rotational bands[1] is one of the most active fields of nuclear structure studies at high spin. More than 300 SD bands have been observed in several mass regions around $A=20$, 40, 80, 130, 150, 165, 190 and 240[2]. Around 200 SD bands were found in $A=80$, 150 and 190 mass regions, in $A=190$ mass region, there are more than 100 SD bands. The intensities of the E2 gamma transitions within a SD band show a remarkable feature: The intra-band E2 transitions follow the band down with practically constant intensity. At some point, the transition intensity starts to drop sharply. This phenomenon is referred to as the decay out of a SD band. It is commonly attributed to a mixing of the SD states and the normally deformed (ND) states with equal(similar)spin. The barrier separating the SD and ND minima of the deformation potential depends on and decreases with decreasing spin. Decay out of the SD band starts from a spin value for which penetration through the barrier is competitive with the E2 decay within the SD band. The decay out of a SD band continues to receive considerable attention of both experiment and theory[3–15].

A precise measurement of the excitation energies and angular momenta of a SD band needs an unambiguous determination of the decay sequences connecting the SD and ND states. Unfortunately, the decay is distributed among many pathways, and the spectrum of gamma rays connecting the SD and ND states is dominated by unresolved transitions. There are rare discrete linking transitions which may be used to determine the excitation energies and spins of SD states. Nevertheless, searching for the discrete linking transition gamma rays is rather difficult and painstaking.

Received date: 1 Sep. 2016
Foundation item: National Natural Science Foundation of China(11275271); Foundation for Innovative Research Groups of the National Natural Science Foundation of China(11321064)
Biography: GU Jianzhong(1963–), male, Suzhou, Jiangsu, Research Fellow, working on nuclear theory;
E-mail: jzgu1963@ciae.ac.cn.

Discrete linking transitions have been identified only in three lead isotopes ^{192}Pb, ^{194}Pb and ^{196}Pb and one mercury isotope ^{194}Hg[8].

There have been many theoretical approaches to study the decay out of a SD band[3, 4, 11, 16–27]. It seems that in theory a consensus has not yet been reached. For instance, the spreading width for the mixing of the SD and ND states has been estimated with different approaches and orders of magnitude difference was found.

Modern nuclear physics allows us to consider nuclear structure problems microscopically. The decay out of a SD band is after all a nuclear structure problem and should be solved based on a microscopic method of nuclear many-body systems. The determination of the barrier separating the SD and ND bands is crucial for the decay out of a SD band. To this end, in the past a few years, we have developed a combined method which fully uses the achievement of modern nuclear theory to calculate the angular momentum projected potential energy (binding energy)surfaces (AMPPES)[11, 12]. The combined method combines a model to describe nuclear ground state properties with another model to describe nuclear excitations which can be used to study nuclear equilibrium shapes, shape coexistence, shape transitions and decay out of SD bands[11].

In this work, with the AMPPES calculated by using the combined method, we study the decay out of SD bands of the lead isotopes ^{190,192}Pb which have been extensively studied both in experiment and theory. The methods used in the calculation are explained in Sec. 2. The numerical results and discussions are presented in Sec. 3. A summary is given in Sec. 4.

2 The combined method based AMPPES and WKB method

Nowadays, there have been many nuclear models and theories, their connections are not so clear. Collecting their merits together one may better understand the nuclear many-body problem. This idea motivates the combined method. During the past decades, the angular momentum projected shell model(PSM)[28–30] has become a standard tool to study nuclear rotational properties up to high-spins. The PSM is a spherical shell model truncated in a deformed basis and solves the many-nucleon system fully quantum mechanically. Because of the advantage of using a deformed basis, the PSM can afford to use a large single-particle space, which ensures that the collective motion and cross-shell excitations are taken into account. Nevertheless, The Hamiltonian of the PSM does not contain the Coulomb interaction of protons which is indispensable for the potential energy surface (PES) of a nucleus. Hartree-Fock-Bogoliubov (HFB) theory (or Relativistic Hartree -Bogoliubov (RHB) theory) which contains the Coulomb interaction successfully describes ground state properties of nuclei, especially binding energies. Combing the PSM and HFB (or RHB), one may obtain the AMPPES up to high spins[11, 12]. The procedure to compute the AMPPES is as follows. We first calculate the ground state PES (bandhead) based on the quadrupole constrained HFB (or RHB)theory. Then, we calculate the PES with a given angular momentum by using the PSM. Finally, we compute the energy difference between the PES with a non-zero angular momentum (spin)given by the PSM and that with zero spin and add the energy difference to the bandhead. Thus, a new PES is then formed, which has a given angular momentum if angular momentum projection is performed for the ground state PES. Those new PESs together with the ground state PES constitute a group of the PESs with given angular momenta. Certainly, if angular momentum projection has not yet been performed for the ground state PES, anything added to the top of it is also unprojected. The ground state binding energy is certain and well defined. It has a natural reference and therefore an absolute physical quantity. Its absolute value is much larger than the excitation energy of interest in nuclear structure, which implies a separation of energy scales and justifies therefore our combined method where nuclear ground states are treated with the HFB(or RHB) and nuclear excitations are described by the PSM[11].

The tunneling probability P for an SD state decaying into a ND state can be calculated by the WKB method,

$$P=\exp\left[-\frac{2}{\hbar}\int_a^b\sqrt{2m(V(x)-E)}\mathrm{d}x\right] , \qquad (1)$$

where m and $V(x)$ are the nuclear mass and potential barrier respectively. The corresponding tunneling width Γ_t for the SD state can be estimated as

$$\Gamma_t=\frac{\hbar\omega_s}{2\pi}P . \qquad (2)$$

$E=\frac{\hbar\omega_s}{2}$ is the zero-point energy in the SD well and is taken to be $E=\frac{1}{2}\hbar\omega_s=0.3$ MeV as was used in Refs. [31, 32] in our calculation.

3 Results and discussion

We combine the PSM with the constrained HFB theory with the Gogny D1S interaction and compute the AMPPES for ^{190,192}Pb from spin $I=0$ up to spin

$I = 22(\hbar)$. The angular momentum projection has been performed for the bandhead[33]. The result is shown in Fig. 1 where $\varepsilon_2 = 0.95\beta_2$ is the quadrupole deformation parameter. One can see from Fig. 1 that the calculated ground states of the two nuclei are spherical, which is consistent with a recent measurement[15]. In addition to a pronounced SD rotational band there are ND rotational bands in ^{192}Pb. In ^{190}Pb, there are ND rotational bands, nevertheless, no pronounced SD band, which is consistent with that fact that no typical SD band has been observed in that nucleus in experiment. In ^{192}Pb, the barrier separating the SD and ND bands becomes lower and lower with decreasing spin.

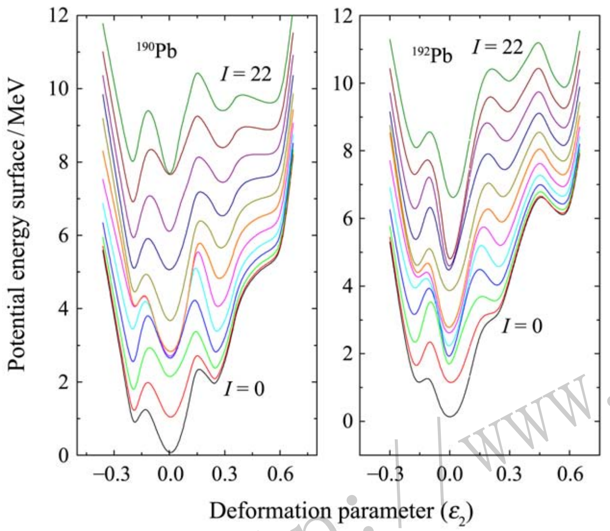

Fig. 1 (color online) The AMPPESs of ^{190,192}Pb calculated with the Gogny D1S interaction, the bandhead is angualr momentum projected.

The decay out of the SD band should be significant. To measure the decay out strength quantitatively, the tunneling width is calculated with the WKB method introduced in the previous section. The values of the tunneling width at different spins are listed in Table 1. The spreading width measures the total strength of the mixing between the SD and ND states, in this sense it should be identical to the tunneling width. The spreading width was estimated with different theoretical approaches for many nuclei in $A = 150$, 190 mass regions in Ref. [5]. The spreading width values of ^{192}Pb at spins 8 and 10 obtained by using the GW approach[3] are consistent with those calculated in the present work, which are quite a few orders of magnitude larger than those given by the SB approach[18]. The tunneling width Γ_t drops sharply at spin 18, indicating the occurrence of the decay out of the SD band at spin 18. That is the SD decay out point of ^{192}Pb.

Table 1 The calculated tunneling width $\Gamma_t(I)$ with various spins I for ^{192}Pb. The tunneling width $\Gamma_t(I)$ is measured in units of eV.

Spin I	Γ_t (I)	Spin I	Γ_t (I)
0	7.9×10^3	12	4.6×10^3
2	7.8×10^3	14	4.0×10^3
4	7.2×10^3	16	2.3×10^3
6	7.2×10^3	18	1.4×10^3
8	6.8×10^3	20	2.8
10	5.0×10^3	22	0.39

The influence of the angular momentum projection on the potential energy surfaces with the Gogny D1S interaction is discussed for ^{190,192}Pb. In the cases with and without angular momentum projection the PESs are calculated and shown in Fig. 2. One can see that the angular momentum projection significantly lowers the barrier separating the SD and ND rotational bands.

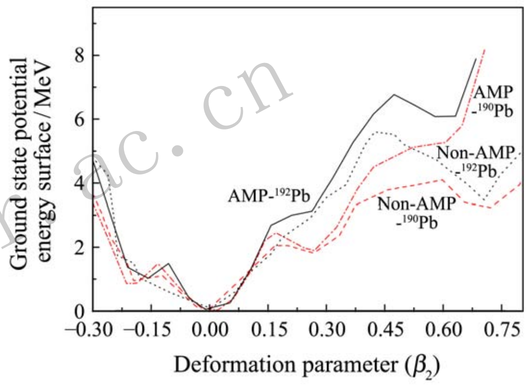

Fig. 2 (color online) Bandheads with the Gogny D1S interaction in the cases with and without angular momentum projection.

The AMPPESs of ^{190,192}Pb are also computed with the most frequently used Skyrme SkP and SLy4 interactions using computer code HFBTHO (v1.66p)[34]. The results are displayed separately in Figs. 3 and 4 where the ground states are also spherical. Pronounced SD rotational bands appear in both nuclei and significantly higher barriers are obtained with the Skyrme interactions compared with those calculated with the Gogny D1S interaction, especially in the case of the SLy4 force. In the case of the Skyrme interactions, the bandheads performed angular momentum projection are similar to those without angular momentum projection[35]. The tunneling width of ^{192}Pb with the Skyrme interactions is still orders of magnitude larger than that given by the SB approach. In fact, in the $A = 190$ mass region the decay from a SD state to the normal states is spread over many different available paths. This means the SD state is coupled to many ND states and the spreading width should be large enough. Therefore, we put the SB approach into question.

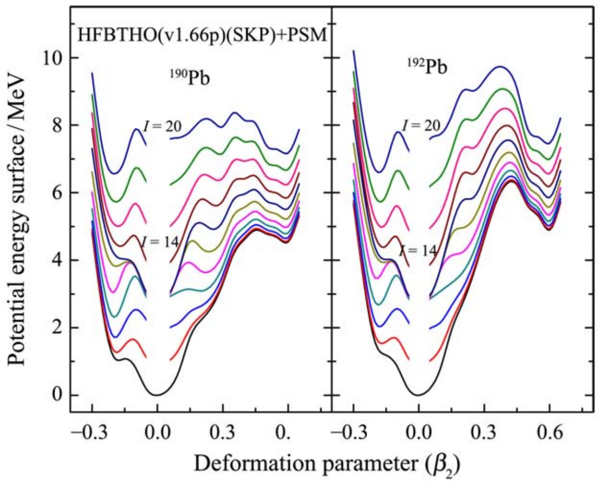

Fig. 3 (color online) The AMPPESs with the Skyrme SkP interaction.

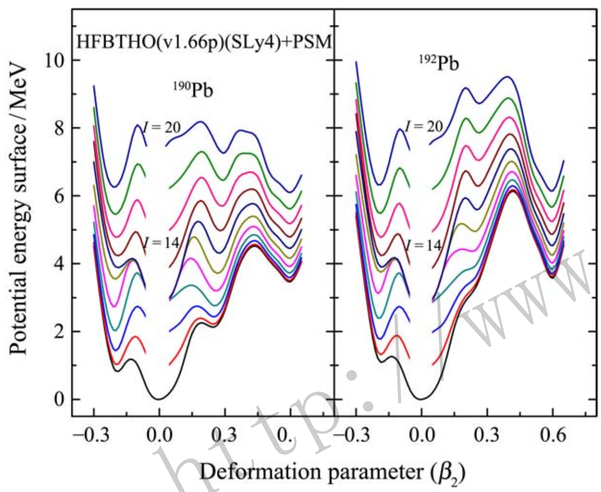

Fig. 4 (color online) Same as in Fig. 3, but with the Skyrme SLy4 interaction.

4 Summary

The SD rotational bands of ^{190,192}Pb have been investigated with the AMPPESs computed with the Gogny D1S and Skyrme SkP and SLy4 interactions. It is shown that there is no typical SD band in ^{190}Pb in the case of the Gogny interaction, which is consistent with the experimental observation. A pronounced SD band with the Gogny interaction appears in ^{192}Pb. The tunneling width of ^{192}Pb is comparable to that given by the GW approach and orders of magnitude larger than that given by the SB approach. The influence of the angular momentum projection on the potential energy surfaces was discussed for ^{190,192}Pb in the case of the Gogny interaction. It is shown that the angular momentum projection lowers the barrier separating the SD and ND rotational bands. Higher barriers of the AMPPESs for the two nuclei computed with the Skyrme SkP and SLy4 interactions were obtained compared with those given by the Gogny force. The tunneling width of ^{192}Pb is big for all of the interactions used in the present work. We therefore put the SB approach into question with which only an extremely small spreading width can be obtained.

References:

[1] TWIN P J, NYAKO B M, NELSON A H, *et al.* Phys Rev Lett, 1986, **57**: 811.

[2] SINGH B, ZYWINA R, FIRESTONe R B. Nucl Data Sheets, 2002, **97**: 241.

[3] GU J Z, WEIDENMÜLLER H A. Nucl Phys A, 1999, **660**: 197.

[4] SARGEANT A J, HUSSEIN M S, PATO M P, *et al.* 2002, Phys Rev C, 2002, **65**: 024302.

[5] WILSON A N, SARGEANT A J, DAVIDSON P M, *et al.* Phys Rev C, 2005, **71**: 034319.

[6] WILSON A N, SINGH A K, HUEBEL H, *et al.* Phys Rev Lett, 2005, **95**: 182501.

[7] WILSON A N, SZIGETI S S, DAVIDSON P M, *et al.* Phys Rev C, 2009, **79**: 014312.

[8] WILSON A N, KORICHI A, SIEM S, *et al.* Phys Rev Lett, 2010, **104**: 162501.

[9] CHIARA C J, SARANTITES D G, MONTERO M, *et al.* Phys Rev C, 2006, **73**: R021301.

[10] WEIDENMÜLLER H A, MITCHELL G E. Rev Mod Phys, 2009, **81**: 539.

[11] ZOU Wenhua, TIAN Yuan, GU Jianzhong, *et al.* Phys Rev C, 2010, **82**: 024309.

[12] GU Jianzhong, PENG Bangbao, ZOU Wenhua, *et al.* Nucl Phys A, 2010, **834**: 87c.

[13] GAFFNEY L P, HACKSTEIN M, PAGE R D, *et al.* Phys Rev C, 2014, **89**: 024307.

[14] BREE N, WRZOSEK-LIPSKA K, PETTS A, *et al.* 2014, Phys Rev Lett, 2014, **112**: 162701.

[15] ESTEVEZ AGUADO M E, ALGORA A, AGRAMUNT J, *et al.* Phys Rev C, 2015, **92**: 044321.

[16] VIGEZZI E, BROGLIA R A, DØSSING T. Nucl Phys A, 1990, **520**: 179C.

[17] SHIMZU Y R, MATSUO M, YOSHIDA K. Nucl Phys A, 2001, **682**: 464c; YOSHIDA K, MATSUO M, SHIMZU Y R. ibid. 2001, **696**: 85.

[18] STAFFORD C A, BARRETT B R. Phys Rev C, 1999, **60**: 051305.

[19] CARDAMONE D, BARRETT B R, STAFFORD C A. Phys Lett B, 2008, **661**: 233.

[20] CARDAMONE D, STAFFORD C A, BARRETT B R. Phys Rev Lett, 2003, **91**: 102502.

[21] BARRETT B R, BUERKI J, CARDAMONE D, *et al*, Phys Lett B, 2003, **688**: 110.

[22] DZYUBLIK A, UTYUZH V. Phys Rev C, 2003, **68**: 024311.

[23] ADAMIAN G G, ANTONENKO N V, JOLOS R V, *et al.* Phys Rev C, 2003, **67**: 054303; ibid. 2004, **69**: 054310.

[24] GU J Z. Int J Mod Phys E, 2008, **17**: 292.
[25] ABERG S, Phys Rev Lett, 1999, **82**: 299.
[26] BERGER J, GIROD M, GOGNY D. Nucl Phys A, 1984, **428**: 32c.
[27] BONCHE P, DOBACZEWSKI J, FLOCARD H, *et al.* Nucl Phys A, 1990, **519**: 509.
[28] HARA K, SUN Y. Int J Mod Phys E, 1995, **4**: 637 .
[29] HARA K, SUN Y, Mizusaki T. Phys Rev Lett, 1999, **83**: 1922.
[30] SUN Y, WALKER P, XU F R, *et al.* Phys Lett B, 2008, **659**: 165.
[31] LAURITSEN T, BENET P, KHOO T L, *et al.* Phys Rev Lett, 1992, **69**: 2479.
[32] KRÜCKEN R, DEWALD A, VON BRENTANO P, *et al.* Phys Rev C, 1996, **54**: 1182.
[33] RODRIGUEZ-GUZMAN R R, EGIDO J L, ROBLEDO L M. Phys Rev C, 2004, **69**: 054319.
[34] STOITSOV M V, DOBACZEWSKI J, NAZAREWICZ W, *et al.* Computer Physics Communication, 2005, **167**: 43.
[35] YAO J M, BENDER M, HEENEN P H. Phys Rev C, 2013, **87**: 034322.

^{190,192}Pb原子核的超形变带的带外衰变

顾建中[1,1)]，彭帮保[2]，李振宇[3]，于国梁[4]

(1. 中国原子能科学研究院，北京 102413;
2. 哈尔滨工程大学核安全与仿真技术国家实验室，哈尔滨 150001;
3. 贵州师范学院物理与电子学院，贵阳 550018;
4. 华北电力大学数理系，河北 保定 071003)

摘要： 将基于组合方法的角动量投影的位能曲面群用于研究铅同位素原子核 ^{190,192}Pb 的超形变转动带。位能曲面群计算中采用了 Gogny D1S 以及 Skyrme SkP SLy4 三种相互作用。如用 Gogny D1S 相互作用，^{190}Pb 原子核没有明显的超形变转动带，而 ^{192}Pb 原子核有非常显著的超形变转动带。用 W. K. B. 方法计算了 ^{192}Pb 原子核的超形变带穿透宽度。该穿透宽度很大与用 GW 模型(Nucl. Phys. A 660 (1999)197)分析实验数据给出的结果相近。在 Gogny D1S 相互作用情形下，就角动量投影对位能曲面群的影响作了讨论，发现角动量投影压低了分隔超形变转动带与正常形变转动带之间的位垒。还用 Skyrme SkP 和 SLy4 相互作用计算了角动量投影的位能曲面群，发现位垒明显高于 Gogny D1S 相互作用给出的位垒。在 Skyrme 相互作用情形下，^{192}Pb 原子核的超形变带穿透宽度明显小于 Gogny 相互作用给出的宽度，但是较SB方法 (Phys. Rev. C, 60 (1999) 051305) 分析实验数据得到的结果高出几个数量级。于是，对 SB 方法提出了质疑,因为它给出了极其微小的传播宽度。

关键词： 组合方法；带外衰变；角动量投影的位能面群

收稿日期： 2016-09-01

基金项目： 国家自然科学基金资助项目(11275271)；国家自然科学基金创新研究群体项目(11321064)

1) E-mail：jzgu1963@ciae.ac.cn。