第34卷 第1期　　原子核物理评论　　Vol. 34, No. 1
2017年3月　　Nuclear Physics Review　　Mar., 2017

Article ID: 1007-4627(2017) 01-0098-07

Nuclear Shape Phase Transitions in SD-pair Shell Model

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Abstract: The nuclear shape phase transitional patterns were studied in the SD-pair shell model. The results show that the transitional patterns similar to the $U(5){-}SU(3)$ and $U(5){-}SO(6)$ transitions in the interacting boson model can be produced. The signatures of the critical point symmetry in the interacting boson model are also produced approximately. It is also found that the shape phase transitional pattern between vibration and rotation can also be produced by changing the interactional strength.

Key words: SD-pair shell model; shape phase transition;spectrum; E2 transution

CLC number: O571.6　　**Document code:** A　　**DOI:** 10.11804/NuclPhysRev.34.01.098

1 Introduction

In the last ten years, a number of theoretical developments have provided new insights on understanding the evolution of nuclear structure in transitional regions through the shape phase transition(SPT) analysis[1–3].

Nuclei, as a mesoscopic system, have been found to possess interesting geometric shapes. Theoretical study of shape phase transitions and critical point symmetries in nuclei has mainly been carried out[2–21] in the interacting boson model(IBM)[2]. The IBM is a phenomenological model of nuclear structure which has a deep connection with the microscopic shell model[22, 23]. Recently there have been studies on nuclear shape phase transitions and their critical point symmetries in the framework of shell model[24–30], density functional approach[31] and relativistic mean field approach[32].

The investigations on nuclear shape phase transition and critical point symmetry for identical nucleon system have also been carried out with fermionic degrees of freedom in[16, 33–37].

Nucleon-pair shell model(NPSM) was proposed in 1993 for even nuclei[38], the advantages of the NPSM are that it accommodates various truncation, ranging from the truncation to only the S subspace, the S-D subspace, up to the full shell model space, and that it is flexible enough to include the broken pair approximation[39], the pseudo $SU(2)$ or the favored pair model[40] and the fermion dynamical symmetry model[41] as its special cases.

The tremendous success of IBM[2], suggests that S and D pairs play a dominant role in the spectroscopy of low-lying modes[42–44]. Therefore, one normally truncates the full shell-model space to the collective SD-pair subspace in the NPSM. The latter is called the SD-pair shell model(SDPSM)[38, 45, 46].

Since the model space is also built up from SD pairs, it is interesting to see if the nuclear shape phase transitional patterns produced from IBM can be produced in the SDPSM. This is the main objective of this paper.

2 Model

In this section we will give a brief description of the SDPSM, while the details of the model can be found in[38, 45].

A schematic Hamiltonian can be adopted in the SDPSM, which is a combination of the single-particle term, monopole pairing, quadrupole-pairing and quadrupole-quadrupole interaction with

$$H=\sum_{\sigma=\pi,\nu}H_\sigma-\kappa_{\pi\nu}Q_\pi^2\cdot Q_\nu^2\ , \tag{1}$$

Received date: 20 Sept. 2016;　**Revised date:** 8 Oct. 2016

Foundation item: National Natural Science Foundation of China(11475091, 11005056, 11175078, 11075080, 11075081, 10935001, 11075052); Natural Science Foundation for Talent Training in Basic Science (J1103208)

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where

$$H_\sigma = \sum_\alpha \epsilon_{\sigma\alpha} n_{\sigma\alpha} - \sum_{j=0,\ 2} G_{\sigma j} S^\dagger_{\sigma j} S_{\sigma j} - \kappa_\sigma Q^2_\sigma \cdot Q^2_\sigma \ ,$$

$$\mathcal{S}^\dagger = \sum_a \frac{\widehat{j}_a}{2} \left(C^\dagger_a \times C^\dagger_a \right) \ ,$$

$$Q^{(2)}_\mu = \sqrt{\frac{16\pi}{5}} \sum_{i=1}^{n} r^2_i Y_{2\mu}(\theta_i \phi_i) \ . \qquad (2)$$

The parameters of $G_{\sigma j}$ and κ_σ are the interaction strength for monopole pairing term,quadrupole-pairing term and quadrupole-quadrupole interaction strength between like-nucleon, and $\kappa_{\pi\nu}$ is the interactional strength of quadrupole-quadrupole between protons and valence neutrons.

The E2 transition operator is

$$E2 = e_\pi Q^2_\pi + e_\nu Q^2_\nu \ , \qquad (3)$$

where e_π and e_ν are effective charge of the neutron and protons.

The "realistic" collective pair of angular momentum S with projection ν, denoted as $A^{r\dagger}_\nu$, is built from many non–collective pairs in the single–particle orbits c and d,

$$A^{r\dagger}_\nu = \sum_{cd} y(cdr)(C^\dagger_c \times C^\dagger_d)^r_\nu, \quad r = 0,\ 2, \qquad (4)$$

where $y(abr)$ are structure coefficients. In this paper, as an approximation, the S-pair structure coefficients are determined as $y(aa0) = \sqrt{2j_a+1}\frac{v_a}{u_a}$, where v_a and u_a are the occupied and unoccupied amplitudes for orbit a obtained by solving the associated BCS equation. The D pair is obtained by using the commutator[47],

$$D^\dagger = \frac{1}{2}\left[Q^2, S^\dagger\right] = \sum_{ab} y(ab2) \left(C^\dagger_a \times C^\dagger_b \right)^2 . \qquad (5)$$

The matrix elements of the Hamiltonian in the multi–pair basis can be expressed in terms of the overlap of the multi–pair states, and the latter can be calculated recursively by[38].

$$\begin{aligned}
&\langle s_1 s_2 \ldots s_N; J'_1 \ldots J'_{N-1} J_N | r_1 r_2 \ldots r_N; J_1 \ldots J_N \rangle = \\
&(\hat{J}'_{N-1}/\hat{J}_N)(-)^{J_N + s_N - J'_{N-1}} \sum_{k=N}^{1} \sum_{L_{k-1} \ldots L_{N-1}} H_N(s_N) \ldots H_{k+1}(s_N) \times \\
&\big[\psi_k \delta_{L_{k-1}, J_{k-1}} \langle s_1 \ldots s_{N-1}; J'_1 \ldots J'_{N-1} | r_1 \ldots r_{k-1}, r_{k+1} \ldots r_N; J_1 \ldots J_{k-1} L_k \ldots L_{N-1} \rangle + \\
&\sum_{i=k-1}^{1} \sum_{r'_i L_i \ldots L_{k-2}} \langle s_1 \ldots s_{N-1}; J'_1 \ldots J'_{N-1} | r_1 \ldots r'_i \ldots r_{k-1}, r_{k+1} \ldots r_N; J_1 \ldots J_{i-1} L_i \ldots L_{N-1} \rangle \big] \ ,
\end{aligned} \qquad (6)$$

where $\hat{J} = \sqrt{2J+1}$, $H_k(s_N)$ are essentially Racah coefficients, induced by various re-coupling procedures, ψ_k is a constant coming from the annihilation of the pair $A^{r_k\dagger}$ by A^{s_N}, and thus depends on the structure of these two pairs, while r'_i represents a new collective pair $B^{r'_i\dagger}$ resulting from a double–process, first the pair A^{s_N} transforms the pair $A^{r_k\dagger}$ into a particle–hole pair $\mathcal{P}^t$ with angular momentum t, which then propagates forward, crosses over the pairs $r_{k-1}, \cdots, r_{i+1}$, and finally transforms the pair $A^{r_i\dagger}$ into the new pair $B^{r'_i\dagger} = \left[A^{r_i\dagger}, \mathcal{P}^t\right]^{r'_i}$, with a new distribution function $y'(a_k a_i r'_i)$ depending on the structure of all the three pairs $A^{r_k\dagger}$, $A^{r_i\dagger}$ and $A^{s_N\dagger}$, and the intermediate quantum numbers $L_i \ldots L_{k-2} L_{k-1}$.

The right side of Eq. (6) is a linear combination of the overlap for N-1 pairs, therefore the overlap can be calculated recursively.

3 Nuclear shape phase transitional patterns as in the IBM

To see if the similar shape phase transitional patterns obtained from the IBM can be reproduced in the SDPSM, the nuclear shape phase transitional patterns for both identical nuclear system and neutron-proton coupled system were studied in the SDPSM. It was found that the results we got from the SDPSM are similar to those from IBM. As an example, the vibration-rotation shape phase transitional patterns for neutron-proton coupled system are presented here.

A schematic Hamiltonian is adopted in the SDPSM, which is a combination of the monopole pairing and quadrupole-quadrupole interaction with

$$\begin{aligned}
H_X = \sum_{\sigma=\pi,\nu} &(-G_\sigma S^\dagger_\sigma S_\sigma - \kappa_\sigma Q^{(2)}_\sigma \cdot Q^{(2)}_\sigma) - \\
&\kappa_\pi \nu Q^{(2)}_\pi \cdot Q^{(2)}_\nu \ ,
\end{aligned} \qquad (7)$$

where X in H_X is denoted as $U(5)$, $SU(3)$ corresponding to vibrational, rotational limiting case in the model, G_σ and κ_σ are the pairing and quadrupole-quadrupole interaction strength between identical-nucleons, respectively. $\kappa_{\pi\nu}$ is the quadrupole-quadrupole interaction strength between proton and neutrons. In this paper, we set $G_\pi = G_\nu$ and $\kappa_\pi = \kappa_\nu$.

To study the phase transitional patterns, the Hamiltonian is written as

$$H = (1-\alpha)H_{U(5)} + \alpha H_X \ , \tag{8}$$

where $0 \leqslant \alpha \leqslant 1$ is a control parameter, H_X is taken as $H_{SU(3)}$ when we study vibration-rotation transitional patterns.

To identify shape phase transitions and determine the corresponding patterns, a set of effective order parameters were proposed, for example, $v_2 = (< 0_2^+|\hat{n}_d|0_2^+ > - < 0_1^+|\hat{n}_d|0_1^+ >)/N$ and $v_2' = (< 2_1^+|\hat{n}_d|2_1^+ > - < 0_1^+|\hat{n}_d|0_1^+ >)/N$[12],$K_1 = B(\mathrm{E2}; 4_1^+ \to 2_1^+)/B(\mathrm{E2}; 2_1^+ \to 0_1^+)$ and $K_2 = B(\mathrm{E2}; 0_2^+ \to 2_1^+)/B(\mathrm{E2}; 2_1^+ \to 0_1^+)$[19],$R_{60} = E_{6_1^+}/E_{0_2^+}$ and $R_{42} = E_{4_1^+}/E_{2_1^+}$.

A system with $N_\pi = N_\nu = 3$ in *gds* shell was studied. By fitting $R_{42} \equiv E_{4_1^+}/E_{2_1^+} = 2$ for vibrational case, $E_{4_1^+}/E_{2_1^+} = 3.33$ for rotational case, the parameters used to produce the vibrational spectra and rotational spectra were obtained, and presented in Table 1. The detailed discussion about the vibrational spectra and rotational spectra can be found in Refs. [49, 50].

Table 1 The parameters used to produce the vibrational, rotational spectra. G_σ is in unit of MeV, κ_σ and $\kappa_{\pi\nu}$ are in unit of MeV/r_0^4.

Limit	G_π	G_ν	κ_π	κ_ν	$\kappa_{\pi\nu}$
Vibration	0.5	0.5	0	0	0.01
Rotation	0	0	0.1	0.1	0.2

Energy ratios R_{42} and R_{60} against control parameter α are shown in Fig. 1. Fig. 1(a) shows that the energy ratio R_{42} is 2 (when $\alpha = 0$) and 3.3 (when $\alpha = 1$), which are typical values of vibrational and rotational spectra, respectively, in the IBM[2]. It is also shown that the rapid change occurs when $0.3 \leqslant \alpha \leqslant 0.6$, which indicates that the phase transition occurs within this region.

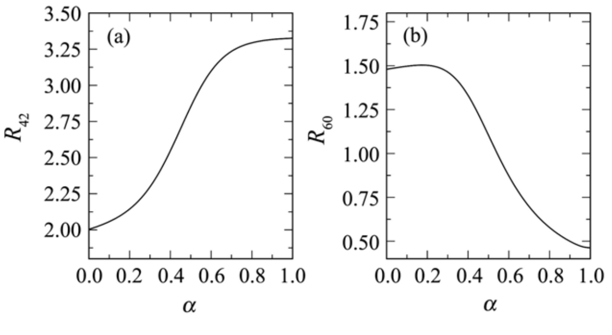

Fig. 1 Energy ratios R_{42} and R_{60} vs α for the vibration-rotation transition.

The energy ratio R_{60} given in Fig. 1(b) shows that similar behavior to that of the IBM for finite number of boson N_B is reproduced. It exhibits a modest peak followed by a sharp decrease across the phase transition, a typical signature of the 1st-order quantum phase transition[52].

The SDPSM results of v_2, v_2', K_1 and K_2 are given in Fig. 2 and Fig. 3. The system we studied here is $A = 130$. The effective charges were fixed with $e_\pi = 3e_\nu = 1.5e$. As argued in Ref. [12], v_2, v_2' should have wiggling behaviors in the region of the critical point due to the switching of the two coexisting phases for the first order phase transition. Indeed, the obvious wiggling behaviors shown by v_2, v_2' in Fig. 2 further confirm that the transition is first order. The results of $B(\mathrm{E2})$ ratio K_1 is also consistent with those of other effective quantities[12, 19]. The critical behavior of K_2 seems to deviate from the character of the first order phase transition.

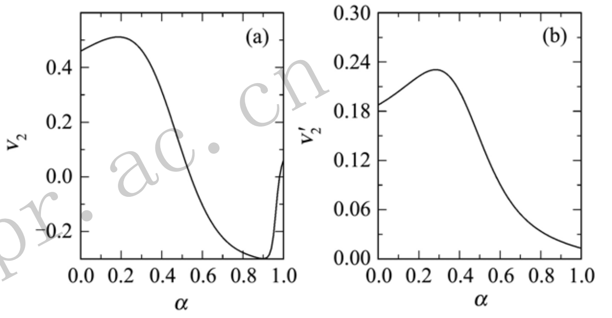

Fig. 2 v_2 and v_2' vs α in the vibration-rotation transition.

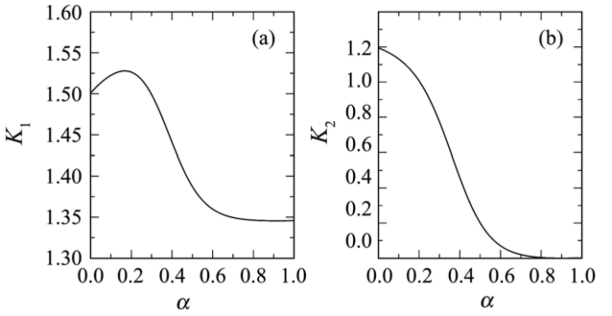

Fig. 3 $B(\mathrm{E2})$ ratios vs α in the vibration-rotation transition.

In the IBM, the critical point symmetry[8] between $U(5)$ and $SU(3)$ is $X(5)$. Since the shape phase transition between vibrational and rotational limit can be reproduced in the SDPSM, it is interesting to see if the properties of the $X(5)$-like symmetry also occurs within the SDPSM. We found that there is indeed a signature with $\alpha = 0.54$ in the SDPSM similar to that of the $X(5)$ in the IBM. A few typical values are given in Table 2, from which one can see that typical feature of the $X(5)$ symmetry stated in Ref. [51, 52] indeed occurs in the SDPSM. For example, R_{42}, R_{60} and $E_{0_2^+}/E_{2_1^+}$ is 2.91, 1.05 and 5.32 in the SDPSM calculation, close to the IBM results 2.91, 1.0 and 5.67, respectively.

Table 2 Energy and B(E2) ratios at vibrational, rotational limit, and $X(5)$-like critical point calculated in the SDPSM.

Limit	$\frac{E_{4_1^+}}{E_{2_1^+}}$	$\frac{E_{6_1^+}}{E_{2_1^+}}$	$\frac{E_{6_1^+}}{E_{0_2^+}}$	$\frac{4_1^+\to 2_1^+}{2_1^+\to 0_1^+}$	$\frac{6_1^+\to 4_1^+}{2_1^+\to 0_1^+}$
Vibrational limit	1.99	2.97	1.47	1.49	1.48
$X(5)$-like point	2.91	5.60	1.05	1.38	1.38
Rotational limit	3.33	6.96	0.46	1.34	1.32
	$\frac{E_{0_2^+}}{E_{2_1^+}}$	$\frac{E_{2^+}-E_{0_2^+}}{E_{2_1^+}}$	$\frac{E_{4^+}-E_{0_2^+}}{E_{2_1^+}}$	$\frac{2^+\to 0_2^+}{2_1^+\to 0_1^+}$	$\frac{4^+\to 2^+}{2_1^+\to 0_1^+}$
$X(5)$-like point (0_2^+ band)	5.32	2.30	5.33	0.37	0.43

4 Effects of interactional strengths on nuclear shape phase transition

In Ref. [53], a correspondence between the strength of each of the interactions and the nuclear shape phases is obtained with the Dyson boson mapping approach(DBMA), in which a shell model Hamiltonian with monopole-pair, quadrupole-pair and quadrupole-quadrupole interactions between nucleons were used. The results show that increasing the quadrupole-pair interaction strength can induce the vibrational to the axially prolate rotational shape phase transition and enhancing the quadrupole-quadrupole interaction can drive the phase transition from the axially oblate rotational to the axially prolate rotational, with the γ-soft rotational being the critical point. Ref. [53] also mentioned that the approximation of bosonization of nucleon pair was applied in their Dyson Boson mapping approach, and whether some results are specifically related to the approximation still needs to be checked. It is interesting to see if the similar conclusion as in Ref. [53] can be obtained in the SDPSM.

The effects of the interactional strength on the nuclear shape phase transitional patterns for both identical nuclear system and neutron-proton coupled system were studied in the SDPSM. It was found that the results we got from the SDPSM are similar to those from DBMA. As an example, the effects of the quadrupole-quadrupole interactional strength on the vibration-rotation transitional pattern for the identical nucleon system are discussed here.

As in Ref. [53], we take a general shell model Hamiltonian to study the dependence of the shape phases on each of the interactions, which is a combination of the single particle energy, monopole pairing, quadrupole-pairing and quadrupole-quadrupole interaction with

$$H = H_0 - G_0\mathcal{S}^\dagger\mathcal{S} - G_2\mathcal{P}^\dagger\mathcal{P} - \kappa Q^{(2)}\cdot Q^{(2)} \ , \qquad (9)$$

$$H_0 = \sum_a \epsilon_a n_a \ ;$$

$$\mathcal{S}^\dagger = \sum_a \frac{\widehat{j_a}}{2}\left(C_a^\dagger \times C_a^\dagger\right) \ ,$$

$$\mathcal{P}^\dagger = \sum_{ab} q(ab2)\left(C_a^\dagger \times C_b^\dagger\right)^2 \ ;$$

$$Q_\mu^{(2)} = \sqrt{\frac{16\pi}{5}}\sum_{i=1}^{n} r_i^2 Y_{2\mu}(\theta_i\phi_i) \ ,$$

where a denote all quantum number necessary to specify a state $[a \equiv (nlj)]$. ε_a and n_a are the single-particle energy and number operator of state a, $\widehat{j}_a = \sqrt{2j_a+1}$ respectively. G_0, G_2 and κ is the monopole-pairing, quadrupole-pairing and quadrupole-quadrupole interaction strength, respectively.

The $E2$ transition operator is simply

$$T(E2) = e_{\mathrm{eff}}Q^{(2)} \ , \qquad (10)$$

where e_{eff} is the effective charge, which is set to be $1.0e$.

To see if the SDPSM can produce the similar results as in Ref. [53], the same major shell and single particle energy levels as in Ref. [53] are used for the system with $A = 130$, which is 1.3, 2.8, 0, 0.8 and 2.5 for $j = 1/2$, $3/2$, $5/2$, $2/7$ and $11/2$, respectively. We set $G_0 = 0.2$ MeV, $G_2 = 0$ and $0 \leqslant \kappa \leqslant 0.1$ MeV/r_0^4, as in DBMA. Since $G_0 = 0.2$ MeV and $G_2 = 0$ corresponds to the spherical phase, our calculation here shows in fact the effect of the quadrupole-quadrupole interaction on the spherical phase. The calculated results of energies, and B(E2) values against κ are given in Fig. 4 and Fig. 5, respectively.

From Fig. 4(a) and Fig. 4(b) one can see that the degenerate level structure of the vibrational states ($U(5)$ symmetric states in the IBM), such as 0_2^+, 2_2^+ and 4_1^+ states, can be produced very well before $\kappa \leqslant 0.01$ MeV/r$_0^4$, while the level structure of the rotational states ($SU(3)$ symmetric states in the IBM) can be re-

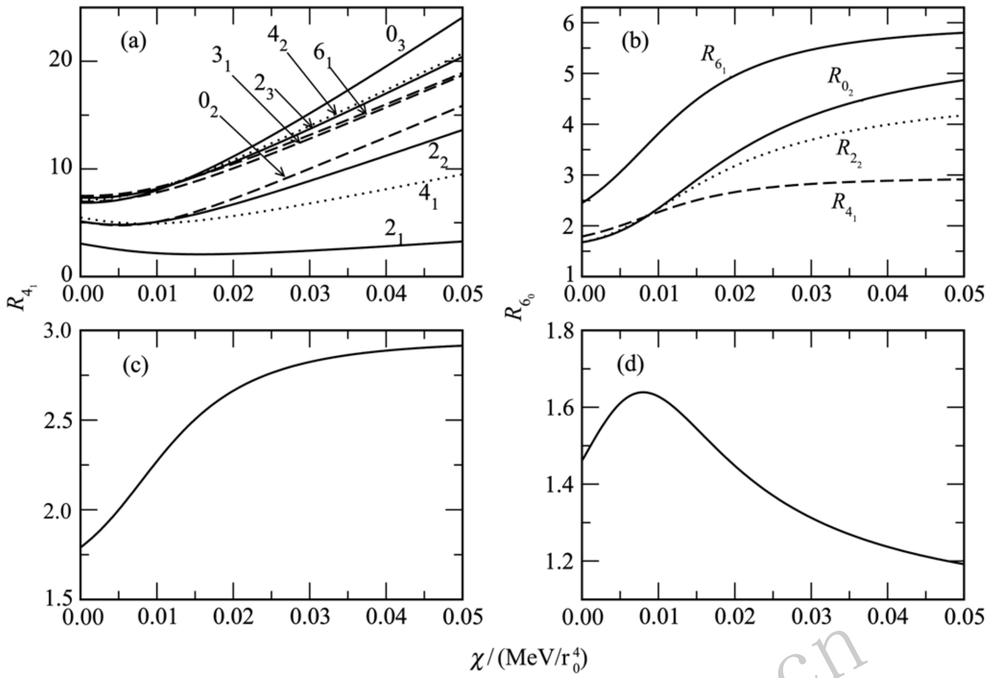

Fig. 4 Calculated result of the dependence of the low-lying levels energies on the quadrupole-quadrupole interaction κ when $G_0 = 0.2$ MeV and $G_2 = 0$. Energy ratios are defined as $R_{J_i} = E_{J_i}/E_{2_1}$, $R_{6_0} = E_{6_1}/E_{0_2}$.

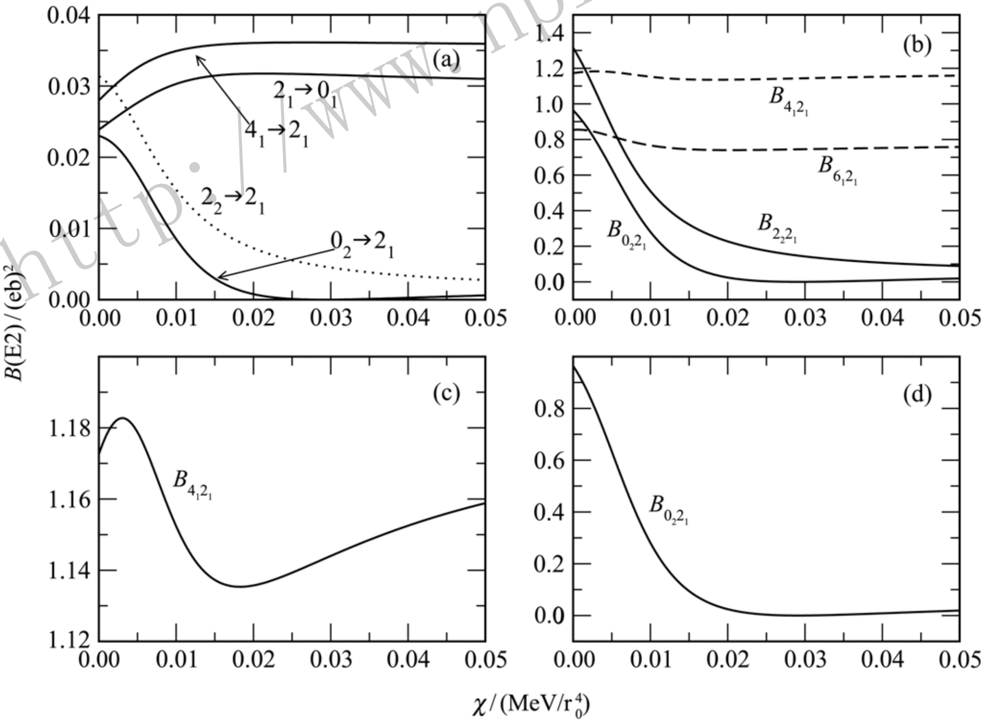

Fig. 5 Calculated result of the dependence of the B(E2) and B(E2) ratios on the quadrupole-quadrupole κ when $G_0 = 0.2$ MeV and $G_2 = 0$. B(E2) ratios are defined as $B_{I_iJ_j} = B(\mathrm{E2}; I_i \to J_j)/B(\mathrm{E2}; 2_1 \to 0_1)$.

produced for larger κ values. The two special energy ratios, R_{4_1} and R_{6_0} which can also be used to identify the nuclear shape phase transition, are presented in Fig. 4(c) and Fig. 4(d), from which one can see that although R_{4_1} is smaller than 2.0 when $\kappa = 0$ and smaller than 3.3 when $\kappa = 0.05$ MeV/r_0^4, the general behavior of the nuclear shape phase transition from vibrational limit to rotational limit can be produced.

For example, Fig. 4(d) shows that the wiggling behavior of R_{6_0} is produced, a typical feature of nuclear shape phase transition between vibrational and rotational limit. The reason why R_{4_1}s are all smaller than the typical value of the vibrational limit 2.0 and rotational limit 3.3 is because the pauli-blocking effect, which plays an important role in producing the collectivity of the low-lying states. If neutron-proton coupled system is considered, the results are all close to the typical values of the limiting cases in the IBM.

The B(E2) and relative B(E2) ratios are given in Fig. 5. From Fig. 5(a) and Fig. 5(b) it is seen that the results of the vibrational limit can be produced when $\kappa = 0$, and then they all change quickly with κ till $\kappa = 0.015$ MeV/r_0^4, after this point, they all become saturate slowly and close to the rotational case, *i.e.*, $B(\mathrm{E2};2_1^+ \to 0_1^+)$ and $B(\mathrm{E2};4_1^+ \to 2_1^+)$ are strong, while $B(\mathrm{E2};2_2^+ \to 2_1^+)$ and $B(\mathrm{E2};0_2^+ \to 2_1^+)$ are small. $B_{4_1 2_1}$ and $B_{0_2 2_1}$ are also used to identify the order of the nuclear shape phase transition[19]. From Fig. 5(c) one can see that the wiggling behavior of $B_{4_1 2_1}$ can be produced, the typical feature of the phase transitional pattern between $U(5)$ and $SU(3)$. But as discussed in Ref. [28], the behavior of $B_{0_2 2_1}$ shown in Fig. 5(d) can not give any signature of the order of the shape phase transition.

5　A brief summary

In summary, the effect of the interactional strength on the nuclear shape phase transition patterns have been studied within the framework of the SD-pair shell model for identical system. The results show that by changing the monopole pairing interactional strength, the nuclear phase from single-particle motion to collective motion can be produced. It is also shown that the shape phase transitional patterns as in the IBM case can also be produced in the SDPSM by changing the interactional strengths. This results also show that the results obtained in Ref. [53] about the validity of the boson mapping is reasonable if the general behavior of the vibration-rotation shape phase transitional patterns are considered.

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SD 对壳模型下原子核形状相变

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摘要： 在SD对壳模型的理论框架下讨论了原子核形状相变模式。研究结果表明，该理论模型可以把相互作用玻色子模型中$U(5)-SU(3)$以及$U(5)-SO(6)$ 形状相变模式再现出来，相互作用玻色子模型中有关临界点对称性的特征也可以很好地描述。本文同时也发现原子核从振动到转动的形状相变可以通过改变相互作用强度来实现。

关键词： SD对壳模型；原子核形状相变；能谱；E2 跃迁

收稿日期： 2016-09-20； **修改日期：** 2016-10-08

基金项目： 国家自然科学基金资助项目(11475091, 11005056, 11175078, 11075080, 11075081, 10935001, 11075052)；国家自然基金委基本科研能力训练项目(J1103208)

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