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# Relativistic Effects in Nuclear Matter with Lattice NN Potential

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**Abstract:** The relativistic effect in nuclear matter is investigated with the latest lattice nucleon-nucleon (NN) potential. A one-boson-exchange potential (OBEP) including three mesons, pion, $\sigma$ meson and $\omega$ meson was constructed based on the lattice NN potential. The meson-nucleon coupling constants and cut-off momentums are determined by fitting the phase shifts of NN scattering from lattice NN potential. The properties of nuclear matter with this OBEP from lattice potential are calculated by one very successful *ab initio* many-body method, Brueckner-Hartree-Fock model. The equations of state and saturation properties of symmetric nuclear matter present very obvious different behaviors in non-relativistic and relativistic frameworks. The relativistic effect plays attractive contributions with the components of S and D waves in lattice NN potential, which is opposite comparing to the relativistic effect from the conventional NN potential.

**Key words:** lattice NN potential; relativistic effect; nuclear matter

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## 1 Introduction

The interaction between two nucleon is always one of the most essential subject in nuclear physics from the discovery of neutron. In the past 80 years, one-boson-exchange model achieved a lot of successes in description on the nucleon-nucleon (NN) potential after Yukawa proposed the pion exchange mode in NN interaction. With the development of quantum chromodynamics (QCD) theory, the NN potential was recognized as a residual effect of the strong force at low energy region, where the QCD theory is non-perturbative. Therefore, the NN potential could not be solved analytically from the QCD theory directly.

Recently decade, the nucleon-nucleon (NN) potential was extracted by Hatsuda *et al.*[1−3] (HAL group) based on the imaginary-time Nambu-Bethe-Salpeter (NBS) wave functions in lattice QCD, where the pion masses from the lattice calculation are generated as five values between 468.6 to 1161.0 MeV[4, 5]. These potentials could present the basic characters of NN interaction more or less, such as strong repulsive core in center channel and attractive tensor component at intermediate region.

These lattice NN potentials were adopted by Inoue *et al.*[6, 7] on the study of nuclear matter and finite nuclei. A saturation point ($\rho_0 = 0.414$ fm$^{-3}$, $E/A = -5.4$ MeV) was obtained for symmetric nuclear matter at the lightest quark mass ($m_\pi = 468.6$ MeV, $M_\mathrm{N} = 1161$ MeV) with a successful microscopic many-body theory, Brueckner-Hartree-Fock (BHF) method[8] and the maximum mass of the neutron star was 0.53 times the solar mass. These result are far from the empirical saturation properties of symmetric nuclear matter and the observations of neutron star, however it still indicated the probability of *ab initio* calculation in nuclear structure from QCD theory.

It has been approved that the relativistic effects play very important roles in the nuclear physics to explain the spin-orbit splitting, pseudospin symmetry, saturation mechanism and so on. As a relativistic version of BHF model, the relativistic Brueckner-Hartree-Fock (RBHF) model can generate the reasonable saturation properties of symmetric nuclear matter, by taking the media effect into the NN potential[9]. The relativistic effect provides a repulsive component on binding energy with a mode of nucleon-antinucleon

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excitation[10].

Therefore, in this paper, the relativistic effects of nuclear matter will be studied within lattice NN potential in RBHF model. The NN potential used in RBHF model should be expressed with spinor to include the nuclear media effect. Therefore, a one-boson-exchange potential (OBEP) will be built firstly based on the present lattice NN potential. Here, we only consider the lattice NN (La469) potential with the lightest quark mass ($m_\pi = 468.6$ MeV, $M_{\mathrm{N}} = 1161$ MeV) which is more close the physical mass of pion.

## 2 Relativistic Brueckner-Hartree-Fock model

In this section, the basic framework of RBHF theory will be presented. The effective potential, $G$-matrix instead of realistic NN potential is obtained by solving the Bethe-Goldstone equation[9]

$$G_{ij}(\boldsymbol{k},\boldsymbol{k}') = V^*_{ij}(\boldsymbol{k},\boldsymbol{k}') + \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} V^*_{ij}(\boldsymbol{k},\boldsymbol{q}) \times \frac{Q_{ij}(\boldsymbol{q},\boldsymbol{P}) G_{ij}(\boldsymbol{q},\boldsymbol{k}')}{2E^*(\boldsymbol{P}/2+\boldsymbol{k}') - 2E^*(\boldsymbol{P}/2+\boldsymbol{q})} \ , \tag{1}$$

where $\boldsymbol{P}$ is the c.m. momentum, $\boldsymbol{k}$, $\boldsymbol{q}$ and $\boldsymbol{k}'$ are the initial, intermediate, and final relative momenta, respectively. The Pauli operator projecting onto unoccupied states is given by $Q$. $E^*$ is the single-particle energy of nucleon in nuclear matter, which is written in terms of kinetic energy and potential energy,

$$E^*_i(\mathrm{p}) = T_i(\mathrm{p}) + U_i(\mathrm{p}) \ , \tag{2}$$

where $U_i(\mathrm{p})$ is a single-particle potential generated by $G$-matrix,

$$U_i(\mathrm{p}) = \Re\Big[ \sum_{q \le k_F^n} \langle pq | G_{in} | pq - qp \rangle + \sum_{q \le k_F^p} \langle pq | G_{ip} | pq - qp \rangle \Big] \ . \tag{3}$$

Here $|p\rangle$ and $|q\rangle$ are nucleon states including single-particle momenta, spin, and isospin index. In many-body system, one nucleon in nuclear medium can be regarded as a 'bare' nucleon that is 'dressed' by its effective two-body potential. Such a 'dressed' nucleon state should satisfy the Dirac equation in relativistic framework,

$$(\not{p}_i - m_i - \Sigma_i(\mathrm{p})) u_i(\boldsymbol{p},s) = 0 \ , \tag{4}$$

where, $\Sigma_i(\mathrm{p})$ is the relativistic self-energy of nucleon and $i$ represents the neutron or proton. The self-energy must have the following general Lorentz structure as symmetry required,

$$\Sigma_i(\mathrm{p}) = U_{S,i}(\mathrm{p}) + \gamma_0 U^0_{V,i}(p) - \boldsymbol{\gamma}\cdot\boldsymbol{p} U_{V,i}(\mathrm{p}) \ , \tag{5}$$

where $U_{S,i}$ and $U_{V,i}$ are an attractive scalar potential and a repulsive vector potential, respectively, and $U^0_{V,i}$ is the time component of vector potential. Actually, $U_{V,i}$ is much smaller than $U_{S,i}$ and $U^0_{V,i}$ as shown in Ref. [9]. Thus we can give the self-energy as

$$\Sigma_i(\mathrm{p}) \approx U_{S,i}(\mathrm{p}) + \gamma_0 U^0_{V,i}(\mathrm{p}) \ . \tag{6}$$

Now, the positive energy solution in above Dirac equation can be obtained,

$$u_i(\boldsymbol{p},s) = \left( \frac{m^*_i + E^*_i(\mathrm{p})}{2m^*_i} \right)^{1/2} \begin{bmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{m^*_i + E^*_i(\mathrm{p})} \end{bmatrix} \chi(s) \ , \tag{7}$$

where $\chi(s)$ is a Pauli spinor, $m^*_i$ the effective nucleon mass and $E^*_i(p)$ effective single-particle energy related with self-energy,

$$m^*_i(\mathrm{p}) = m_i + U_{S,i}(\mathrm{p}) \tag{8}$$

and

$$E^*_i(\mathrm{p}) = (m^{*2}_i + \boldsymbol{p}^2_i)^{\frac{1}{2}} \ . \tag{9}$$

Due to the spinor states, the single-particle potential can be formed from the Dirac equation,

$$U_i(\mathrm{p}) = \frac{m^*_i}{E^*_i} \langle p | U_{S,i} + \gamma^0 U^0_{V,i} | p \rangle \ , \tag{10}$$

where $|p\rangle$ is a Dirac spinor and $\langle p|$ is the conjugate spinor. $U_{S,i}$ and $U^0_{V,i}$ are treated as constants at a fixed density as an approximation[9]. Thus, the single-particle potential is simplified as,

$$U_i(\mathrm{p}) = \frac{m^*_i}{E^*_i(\mathrm{p})} U_{S,i} + U^0_{V,i} \ . \tag{11}$$

$U_{S,i}$ and $U^0_{V,i}$ will be determined from the Eq. (3) self-consistently. Later, the energy per neutron or proton in nuclear matter is obtained through the relativistic Hartree-Fock wave functions.

## 3 Results and discussion

Firstly, we construct a OBEP from the data of La469 potential. Three mesons, pion, σ meson, and ω meson are considered in present OBEP. Pion can provide the tensor force and lang range part of NN potential, while the σ meson and ω meson are the original sources of middle-range and short-range contribution of NN potential respectively. Their coupling constants with nucleon and cut-off momentums will be determined by fitting the phase shifts calculated by La469 potential, named LOBEP at $^1S_0$, $^3S_1$, $^3S_1$–$^3D_1$ channels. We tabulate the meson-nucleon coupling constants and cut-off momentums of LOBEP potential in Table 1. The masses of pion, ω meson and nucleon have been calculated in lattice QCD. The σ meson mass is a free parameter.

Table 1　The meson parameters of LOBEP, where the nucleon masses are taken as $M_{\mathrm{N}} = 1161.0$ MeV from lattice calculation.

|  | $m_\alpha$ /MeV | $g_\alpha^2/4\pi$ | $\Lambda_\alpha$/MeV |
|---|---|---|---|
| π | 468.6 | 17.00 | 902.78 |
| σ | 491.9 | 8.56 | 699.27 |
| ω | 829.2 | 12.76 | 1129.03 |

The phase shifts of LOBEP potential are given in the upper panel of Fig. 1 at $^1S_0$, $^3S_1$, and $^3S_1-{}^3D_1$ channels. In the lower panel, $\varepsilon_1$ is the mix parameter of $^3S_1-{}^3D_1$ for coupled states. These phase shifts of LOBEP potential accord to ones from La469 potential very well, whose $\chi^2/N_{\mathrm{data}} \sim 0.2$ up to laboratory energy $E_{\mathrm{lab}} = 300$ MeV.

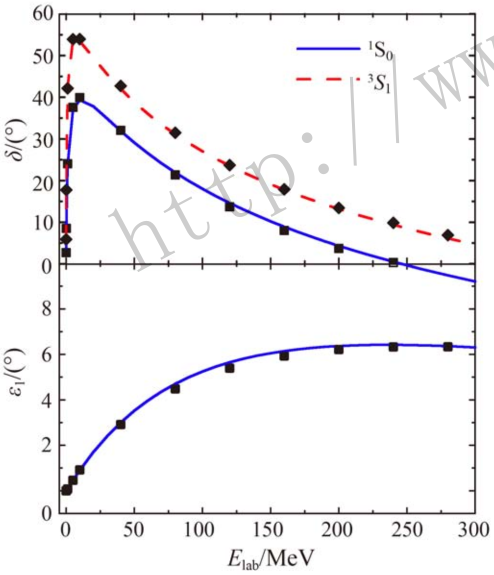

Fig. 1　(color online) The phase shifts of LOBEP potential and La469 potential at different channels. The solid curves represent the phase shifts of LOBEP, while the symbols are the ones of La469 potential.

Now, with the LOBEP potential, the equation of state (EOS) of nuclear matter can be worked out with BHF theory only taking the $^1S_0$, $^3S1$, $^3S_1-{}^3D_1$ and $^3D_1$ channels. The EOSs of symmetric nuclear matter ($\delta = \dfrac{N-Z}{A} = 0$) in BHF and RBHF theory are given in Fig. 2. In BHF theory, the binding energy $E/A = -5.47$ MeV at saturation density $\rho = 0.39$ fm$^{-3}$ for LOBEP potential, which are consistent with the results by Inoue *et al.*[6]. Because the calculation precesses of phase shifts and BHF model are solving the Lippmann-Schwinger-type equations with both on-shell and off-shell matrix elements of NN potential. The saturation properties of symmetric nuclear matter are $E/A = -12.34$ MeV at $\rho = 0.63$ fm$^{-3}$. It looks that the non-relativistic case provides more repulsive effect with present lattice NN potential, which is an opposite conclusion comparing with the previous RBHF model with Bonn A potential[9]. But we should remember that there are only $L = 0, 2$ channels available in present lattice potential. We do not have the data of lattice potential at $L = 1$ channels.

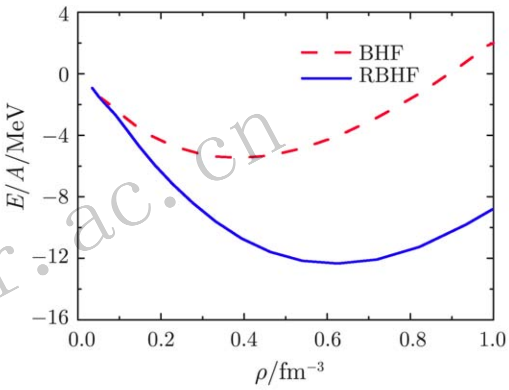

Fig. 2　(color online) The EOSs of symmetric nuclear matter with BHF and RBHF theories for LOBEP potentials. The BHF results are shown in dashed curve. The RBHF ones are given in solid line.

## 4　Conclusions

A one-boson-exchange potential (OBEP) was constructed based on the latest lattice NN potential (La469) to study the properties of nuclear matter with relativistic Brueckner-Hartree-Fock theory. We fitted the OBEP with the phase shifts of La469 potential and obtained LOBEP potential, which could completely reproduce the fitting data. The saturation properties of this OBEP in Brueckner-Hartree-Fock theory were consistent with the existing calculation by Inoue *et al.* for La486 potential directly including $^1S_0$, $^3S_1$, $^3S_1-{}^3D_1$ and $^3D_1$ channels. There were more attractive contributions for saturation energy in RBHF theory. This result is opposite with the conventional calculation in RBHF theory with Bonn potential, which is obtained by the nucleon-nucleon scattering data. We hope that the lattice QCD can provide more data of NN potential and decrease the quark mass to approach the practical pion mass so that we can realize the *ab initio* calculation of nuclear many-body system from QCD level.

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# 格点核子-核子势在核物质中的相对论效应

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**摘要：** 利用最新的格点核子-核子势研究了核物质中的相对论效应。通过此格点核子-核子势场，首先我们构建一个包括π介子，σ介子以及ω介子的单玻色子交换势。势场中的介子-核子耦合常数以及截断动量通过拟合格点核力得到的核子-核子散射相移确定。随后采用非常成功的第一性原理多体计算方法Brueckner-Hartree-Fock模型，计算了核物质的基本性质。发现对称核物质的状态方程以及饱和性质在非相对论框架和相对论框架中有很明显的区别。在格点核力中，该相对论效应对核物质的结合能提供吸引的贡献。这与采用传统的核力计算得到的结果是相反的。

**关键词：** 格点核子-核子势；相对论效应；核物质

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