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Regge-like Masses of Light Mesons in Quark and String Models

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Abstract: We revisit the Regge-like spectra of light mesons in the relativistic string model and the simi-relativistic quark model. An analytical mass formula is proposed for light mesons based on the auxiliary field technique, and a quasi-linear Regge-Chew-Frautschi plot with flavor dependence is derived and verified with the experimental data(PDG) for light mesons.The results show that the quark model predictions for meson masses agree with those of the string model, but are slightly better when the angular momentum is relatively large as compared with the observed data.

Key words: light meson; Regge trajectory; quark model; string model

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1 Introduction

The spectra and Regge trajectories of mesons has been recurrent subject of strong interaction. Analysis of Regge trajectories is of interest as it manifests not only the underlying physics of QCD dynamics at long distance, which becomes dominated by the string-like configurations that leads to confinement, but also it may provide a clue to identify extra (exotic) states among a wide range of heavy mesonic resonances discovered in recent years.

It is remarkable experimentally that most hadrons which consist of light quarks can be grouped in rotational families according to a simple linear relation (Regge-Chew-Frautschi plot) between orbital angular momentum (J) and mass squared[1]

$$M^2 = (2\pi T_0)J + a \ . \qquad (1)$$

In the case of mesons (which we shall discuss in this work), though the intercept a may differ for different hadrons, but almost all hadrons fall,in the plot of (J, M^2) plane, on various straight lines known as "Regge trajectories". There are some discussions if Regge trajectories are linear, parallel, or not[2, 3], but without systematic errors included. So, the linearity is usually assumed.

Linearity of Regge trajectories for the light mesons was first explained in the model of rotating relativistic string [4–6]. The flavor dependence of Regge trajectory was discussed [7]. The linear trajectories and their nonlinear improvement of light mesons have been studied in Refs. [8–13].

In the picture of AdS/QCD, Forkel *et al.*[14, 15] predicted a Regge-like mesonic mass relation(see also Ref. [16]

$$M^2 = 4\lambda^2(n + J + 1/2) \ , \qquad (2)$$

for the ground state of light mesons. However, Regge trajectories remain to be understood in quark models. For further works see Refs. [17-20] and references therein.

We revisit the spectra and Regge trajectories of the light mesons in the relativistic model of string and semi-relativistic quark model. An analytical mass formula for light mesons is proposed for the limit of the large $N = n + J$ with n the radial quantum number and J the orbital quantum number based on the auxiliary field(AF) technique, and a quasi-linear Regge-Chew-Frautschi (RCF) plot is derived, in which the flavor dependence arises from the quark masses. The derived RCF plot is verifiled with the observed data of the light mesons in (M^2, J) plane, with a good agreement with the experimental spectra.

2 Mesonic mass in string model

A native mass formula that simply follows from

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Eq. (1) is

$$M=\sqrt{J/\alpha'}+\frac{a}{2\sqrt{J/\alpha'}}\ , \qquad (3)$$

which naturally leads to Eq. (1) when J becomes large or a is very small. Here $\alpha'=1/(2\pi T_0)$ and T_0 is the tension of string. Assuming the picture of rotating string[4–6], with the almost massless quark and anti-quark at the ends of string, one has roughly for the length of string

$$l=M/T_0\ . \qquad (4)$$

Combining with Eq. (1) with a ingored, it follows that

$$l=\sqrt{2\pi J/T_0}=2\pi\sqrt{\alpha' J}$$

and a mass relation in terms of the string length, from Eq. (3), becomes

$$M=T_0 l+\frac{a}{2T_0 l}\ . \qquad (5)$$

This is in constistent with the energy spectrum of the free bosonic string[21],

$$E_n(l)=T_0 l+\frac{c}{l}+\mathcal{O}(l^{-3}), \text{for } l\gg 1/\sqrt{T_0}\ . \qquad (6)$$

The flavor dependence of Eq. (3) can be incorporated in the relativistic model of string, by loading the quark mass($m_{1,2}=m$) to the ends of string, for which the energy and the orbital angular momentum are

$$E=\frac{2m}{\sqrt{1-v^2}}+T_0\int_{-l}^{l}\frac{\mathrm{d}s}{\sqrt{1-(\omega s)^2}}\ ,$$
$$J=\frac{2m\omega l^2}{\sqrt{1-v^2}}+T_0\omega\int_{-l}^{l}\frac{s^2\mathrm{d}s}{\sqrt{1-(\omega s)^2}}\ , \qquad (7)$$

where $v=\omega l$ with ω denoting the angular velocity of rotating string. The bondary condition at the ends $s=\pm l$ can be given by the equation of motion of E in Eq. (7): $T_0 l=mv^2/(1-v^2)$, from which Eq. (7) simplifies

$$E=\frac{2m}{\sqrt{1-v^2}}\left[v\arcsin(v)+\sqrt{1-v^2}\right]\ ,$$
$$J=\frac{m^2}{T_0\sqrt{1-v^2}}\left[\arcsin(v)+v\sqrt{1-v^2}\right]\ . \qquad (8)$$

Notes that $v=(1+\epsilon_m^2/4)^{1/2}-\epsilon_m/2\simeq 1-\epsilon_m/2+\epsilon_m^2/8$, with $\epsilon_m\equiv m\omega/T_0$, which means the velocity of the string ends $v\to 1$(speed of light) when $m\sim\epsilon_m\to 0$. Since ϵ_m is small for the light mesons, one can show, by expanding in ϵ, that

$$\frac{m}{E}\simeq\frac{\epsilon_m}{\pi}-\frac{2\epsilon_m^{5/2}}{3\pi^2}+\cdots,$$
$$\frac{J}{E^2}=\frac{1}{4T_0}\left[\frac{2}{\pi}-\frac{16\epsilon_m^{3/2}}{3\pi^2}+\cdots\right]\ . \qquad (9)$$

Combining two of equations Eq. (9) yields

$$J=\alpha' E^2\left[1+\frac{8\sqrt{\pi}}{3}\left(\frac{m}{E}\right)^{3/2}\right]$$

which, solved for E, gives

$$E^2=\frac{J}{\alpha'}+\frac{8}{3}\sqrt{\pi}m^{3/2}\sqrt[4]{\frac{J}{\alpha'}}+\mathcal{O}(m^3)\ . \qquad (10)$$

We note that a full quantum mechanical treatment of the string[22] provides the "quantum defect" a in Eq. (1). With a added, Eq. (10) agrees with Eq. (1) when $m\to 0$. Rewriting Eq. (10) into the form of Regge-like relation, it becomes

$$E^2=2\pi T_0 J\left(1+\frac{8\sqrt{\pi}}{3}\left(\frac{m}{\sqrt{2\pi T_0 J}}\right)^{3/2}\right)+a\ . \qquad (11)$$

We conclude this section by giving the following remarks: (i) The mesonic mass squared gets a correction of $\sqrt[4]{J}$ when the quark mass is added to the string ends; (ii) the quantum correction is necessary for reproducing the Regge trajectory Eq. (1); (iii) The trajectory slope $\alpha'=1/(2\pi T_0)$ becomes smaller slightly by a factor of $\left(1+\frac{8}{3}\frac{\sqrt{\pi}m^{3/2}}{(2\pi T_0 J)^{3/4}}\right)$, which depends on the $m/\sqrt{T_0 J}$.

3 Quark model for light mesons

To discuss the Regge trajectory of light mesons in quark model, we use a semi-relativistic Hamiltonian in which the kinematic Hamiltonian is that of the Godfrey-Isgur (GI) quark model[23]. Differing from the later, the spin-dependent interactions are ignored for simplicity. The model Hamiltonian is

$$H=\sum_{i=1}^{2}\sqrt{\boldsymbol{p}^2+m_i^2}+V\ , \qquad (12)$$

where V is the color-confining potential with the form of the linear plus Coulomb interaction

$$V=Tr-\frac{4}{3}\frac{\alpha_s}{r}+V_0 \qquad (13)$$

with T the string tension, V_0 a constant, and α_s the strong coupling constant between the quark and anti-quark. In Eq. (13), and $r=|\boldsymbol{x}_1-\boldsymbol{x}_2|$ is the relative coordinate of the quark 1 and anti-quark 2, with the masses m_1 and m_2, respectively.

Before solving the model Hamiltonian Eq. (12), let us consider the ultrarelativistic limit $m_{1,2}=m\to 0$, and the case that the color-Coulomb interaction is switched off ($\alpha_s = 0$), for which the Hamiltonian (12) simplifies greatly

$$H = 2p + T|r| + V_0 \ . \tag{14}$$

This corresponds to the simple case that the energy of whole system is given by that of a string with massless ends: $E^2 \propto J$[22]. Here, to treat the boundary condition at $r = 0$, we extend the radial space $(0, \infty)$ of r to $(-\infty, \infty)$, by writing potential as $T|r| + V_0$. The turning points for Eq. (14) are $r^{\pm} = \pm(M - V_0)/T$, and the WKB quantization condition gives

$$n\pi = \frac{1}{2}\int_{r^-}^{r^+} (M - T|r| - V_0)\mathrm{d}r = \frac{(M - V_0)^2}{2T} \ , \tag{15}$$

that is,

$$(M - V_0)^2 = 2\pi T n \ .$$

This gives a radial Regge relations $M^2 = 2\pi T n$ when $V_0 = 0$. It implies that the ratio of the slope of radial orbital is about $1:1$, as suggested by Afonin[16]. When combining with Eq. (1) and choosing $T = T_0$, it yields a combined trajectory relation $M^2 \propto n + J$, as predicted in Eq. (2).

To solve Eq. (14) quantum-mechanically, we use the auxiliary field (AF) technique [9, 11, 24] (using the inequality $\sqrt{B} \leqslant \dfrac{B}{2\lambda} + \dfrac{\lambda}{2}$, where equality holds only if $\lambda = \sqrt{B}$, λ is positive) to rewrite the model Hamiltonian (12) as $H = \min_{\mu_{1,2},\nu}\{H(\mu_{1,2},\nu)\}$, where

$$H(\mu_{1,2},\nu) = \sum_{j=1}^{2}\left[\frac{\mathbf{p}^2 + m_j^2}{2\mu_j} + \frac{\mu_j}{2}\right] + \frac{T^2 r^2}{2\nu} + \frac{\nu}{2} - \frac{4}{3}\frac{\alpha_s}{r} + V_0 \ , \tag{16}$$

and the auxiliary fields, denoted as $\mu_j (j = 1,2)$ and ν, are operators. $H(\mu_j, \nu)$ is equivalent to Eq. (12) up to the elimination of the field $(\mu_{1,2}, \nu)$ with the help of the constraints

$$\begin{aligned} \mu_i \to \mu_{i,0} &= \sqrt{\boldsymbol{p}_i^2 + m_i^2} \\ \nu_i \to \nu_{i,0} &= T|\boldsymbol{x}_1 - \boldsymbol{x}_2| \ . \end{aligned} \tag{17}$$

Here, the state average $\langle \mu_{i,0} \rangle$ $(i = 1,2)$ can be viewed as the dynamical mass of the quark i, and $\langle \nu_{i,0} \rangle$ as the static energy of the flux-tube (QCD string) linking the quark 1 and 2[10, 26].

Treating the auxiliary fields, which are operators, as real c-numbers[10, 12, 26], one can transform Eq. (14) into a problem of quantum harmonic oscillator. In the center of mass system, the Hamiltonian Eq. (16) becomes

$$H(\mu_{1,2},\nu) = \frac{\boldsymbol{p}^2}{2\mu} + \frac{1}{2}\mu\omega^2 r^2 + \frac{\mu_M + \nu}{2} - \frac{A}{2\nu} + \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + V_0 \ , \tag{18}$$

in which $\mu_M = \mu_1 + \mu_2$, $\mu = \mu_1\mu_2/\mu_M$ is the reduced mass of quarks, $A = \dfrac{8}{3}\alpha_s T$, and $\omega = \dfrac{T}{\sqrt{\mu\nu}}$. In deriving Eq. (18), we have estimated the contribution of the color-Coulomb term by its quantum average:

$$-\frac{4}{3}\left\langle \frac{\alpha_s}{r} \right\rangle \approx -\frac{4\alpha_s/3}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} = -\frac{A}{2\nu} \ , \tag{19}$$

where the relation $|\langle|\boldsymbol{x}_1 - \boldsymbol{x}_2|^2\rangle \approx (\nu/T)^2$ (using Eq. (17)) has been used. Since the first line of Eq. (18) is simply the Hamiltonian of the harmonic oscillator, one can diagonalize the Hamiltonian $H(\mu_{1,2},\nu)$ in Eq. (18) in the harmonic oscillator basis $|nJm\rangle$. The result for the quantized energy $E_N(\mu_{1,2},\nu) = \langle|H(\mu_{1,2},\nu)|\rangle_{nJm}$ is

$$E_N(\mu_{1,2},\nu) = \omega\left(N + \frac{3}{2}\right) - \frac{A}{2\nu} + \frac{\mu_M + \nu}{2} + \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + V_0 \ , \tag{20}$$

where $N = n + J + 3/2$, with n and J the radial and orbital quantum numbers, respectively. We note here that as discussed for the limit case Eq. (14), an appropriate counting of the nodes yieds a replacement $2n \to n$ which is taken in Eq. (20) due to the superfluous mirror symmetry of the transformed Hamiltonian Eq. (18) (under $r \to -r$).

4 Mass formula and Regge trajectory

According to the AF method, one has to minimize the energy Eq. (20) so that the three auxiliary fields appearing in Eq. (20) are eliminated. This amounts to solving simultaneously $\partial_w E_N(\mu_{1,2},\nu) = 0 (w = \mu_1, \mu_2, \nu)$, which are

$$\frac{a_N \nu}{2\sqrt{(\mu\nu)^3}}\left(\frac{\mu_2}{\mu_M} - \frac{\mu_1\mu_2}{\mu_M^2}\right) = \frac{1}{2} - \frac{m_1^2}{2\mu_1^2} \ , \tag{21}$$

$$\frac{a_N \nu}{2\sqrt{(\mu\nu)^3}}\left(\frac{\mu_1}{\mu_M} - \frac{\mu_1\mu_2}{\mu_M^2}\right) = \frac{1}{2} - \frac{m_2^2}{2\mu_2^2} \ , \tag{22}$$

$$\frac{a_N \mu}{\sqrt{(\mu\nu)^3}} = 1 + \frac{A}{\nu^2} \ , \tag{23}$$

where we denote $a_N \equiv T(n + J + 3/2)$.

For the light meson (made of s, u or d quarks) for which the quark masses $m_{1,2}$ are small, one can assume $A/a_N \sim \alpha_s/N \ll 1$ which is valid when N is large. One can then solve the equations Eq. (21) through Eq. (23) to obtain (after some algebra)

$$\mu_0=\frac{\mu_{0M}}{4},\mu_{0M}=\frac{4a_N^2\nu_0}{(\nu_0^2+A)^2}\ , \tag{24}$$

$$\mu_1\simeq\frac{\mu_{0M}}{2},\mu_2=\frac{\mu_{0M}}{2}\ , \tag{25}$$

$$\nu_0=\sqrt{2a_N+A_m}\ , \tag{26}$$

where we have used the notation $A_m=\frac{A}{6}-(m_1^2+m_2^2)$.

With the relations for ω, Eq. (24) through Eq. (26), one obtains, from Eq. (20), a mass formula for the light mesons,

$$M(\bar{\mathrm{q}}\mathrm{q})=\left(\frac{3}{2}+\frac{1}{2\chi_N^2}\right)\nu_0+\frac{1}{2\nu_0}\left(A+4\bar{m}^2\kappa_N^2\right)+V_0\ , \tag{27}$$

where $\bar{m}^2=(m_1^2+m_2^2)/2$ and

$$\begin{aligned}\chi_N&=1+\frac{A+A_m}{2a_N}=1+\frac{14\alpha_\mathrm{s}/9-\bar{m}^2/T}{(n+J+3/2)}\ ,\\ \kappa_N&=1+\frac{A_m}{2a_N}=1+\frac{2\alpha_\mathrm{s}/9-\bar{m}^2/T}{(n+J+3/2)}\ .\end{aligned} \tag{28}$$

Since $\nu_0\sim\sqrt{2TJ+A_m}$, the form of Eq. (27) is similar to Eq. (3) which is argued in the sring model before.

Squaring Eq. (27), one gets a simple relation between mass squared (M^2) and the quantum number ($N=n+J+3/2$) which is of the quasi-linear form of the RCF plot

$$M^2=2Tw^2(n+J+3/2)+a^{(0)}\ , \tag{29}$$

in which

$$w=\frac{3}{2}+\frac{1/2}{\left(1+\dfrac{14\alpha_\mathrm{s}/9-\bar{m}^2/T}{(n+J+3/2)}\right)^2}\ , \tag{30}$$

and a_0 is a intercept constant. Since a_0 contains complex contributions from the effects of the vaccum and bound states which goes beyond the simple dynamics of the quark model, it will be fitted to the data in this work.

To confront Eq. (29) with the observed data, we write the slope explicitly as below,

$$\alpha'=1/\left[2T\left(\frac{3}{2}+\frac{1/2}{\left(1+\dfrac{14\alpha_\mathrm{s}/9-\bar{m}^2/T}{(n+J+3/2)}\right)^2}\right)^2\right]\ . \tag{31}$$

We conclude this section as belows: (1) a Regge-like mass relation Eq. (1) can arise from the semi-relativistic quark model: $M^2\propto(n+J)$; (2) The trajectory slope decreases very slowly with N, and the intercept $a^{(I)}$ increases slowly with N; (3) The trajectory is almost linear for small quark mass m; (4) The trajectory slope is close to that of the massless sring $1/(2\pi T)$ for some moderate values of α_s, $m_{1,2}$, T, as discussed in Section 5.

5 Results and discussion

We use Eq. (29), with w given by Eq. (30), to fit the light mesons and find that they can be grouped into the four families according to (π/b, ρ/a, η/h, ω/f) in the (J, M^2) plane (we set $n=0$ in this fit). The optimal parameters (T,α_s,$\bar{m}$, $a^{(0)}$) fitted to the data are shown in Table 1. We also show the global values of the slope α' dertermined by Eq. (31) at the first and last points in the (J, M^2) plane and the intercept in the unit of GeV^2 given by $M^2(n+J=0)$ in Eq. (29). Num. stands for the number of the data coordinates. In Table 1, the calculated masses are compared to that of the experiment(PDG)[25] for the light mesons, with χ^2 between them listed also. Table 2 shows the optimal values for the string tension(T), the strong coupling(α_s) and the quark masses, fitted to the

Table 1 The listed are calculated masses and that of the experiment(PDG)[25] for the light mesons. The quality of comparison is given by the values of χ^2 between the observed and calculated values. They are 0.0015(π/b), 0.0057(η/h), 0.0022(ρ/a), 0.0007(ω/f), respectively, for the mesonic formula predictions.

Mesons	J^{PC}	states	m(Th)	m(String)	m(Exp)
π/b	1^{+-}	$b_1(1235)$	1.259	1.257	1.299
	2^{-+}	$\pi_2(1670)$	1.663	1.650	1.672
	3^{+-}	$b_3(2030)$	1.992	1.965	2.030
	4^{-+}	$\pi_4(2250)$	2.276	2.236	2.250
η/h	0^{-+}	$\eta(548)$	0.4700	0.545	0.5479
	1^{+-}	$h_1(1170)$	1.231	1.206	1.170
	2^{-+}	$\eta_2(1645)$	1.698	1.612	1.617
	3^{+-}	$h_4(2025)$	2.069	1.933	2.025
	4^{-+}	$\eta_4(2330)$	2.384	2.208	2.330
ρ/a	1^{--}	$\rho(770)$	0.8071	0.776	0.7752
	2^{++}	$a_2(1320)$	1.322	1.324	1.318
	3^{--}	$\rho_3(1690)$	1.692	1.701	1.689
	4^{++}	$a_4(2050)$	1.996	2.008	1.995
	5^{--}	$\rho_5(2350)$	2.261	2.274	2.350
	6^{++}	$a_6(2450)$	2.499	2.511	2.450
ω/f	1^{--}	$\omega(782)$	0.7775	0.768	0.7827
	2^{++}	$f_2(1270)$	1.301	1.319	1.275
	3^{--}	$\omega_3(1670)$	1.675	1.698	1.667
	4^{++}	$f_4(2050)$	1.981	2.006	2.018
	5^{--}	$\omega_5(2250)$	2.246	2.271	2.250
	6^{++}	$f_6(2510)$	2.485	2.509	2.510

Table 2　The parameters fitted in quark model.

Mesons	Num	T/GeV^2	α_s	$\bar{m}$/GeV	a^0/GeV^2	α'/GeV^{-2}
π/b	4	0.1548	0.5395	0.1666	−0.9577	0.8349
ρ/a	6	0.1429	0.5395	0.1851	−1.7295	0.8998
η/h	5	0.1799	0.5395	0.1667	−1.4257	0.7194
ω/f	6	0.1423	0.5395	0.1851	−1.7658	0.9040

experimental(PDG) mass data[25]. The calculated trajectory parameters are also listed here.

We see that the mass formula Eq. (29) gives a good agreement with the observed data at least for the low-lying light mesons (π/b, ρ/a, η/h, f/ω). A global optimal values for the trajectory parameter can be given by

$$T=0.155, \alpha_{\mathrm{s}}=0.540, \bar{m}=0.176, \alpha'=0.840$$

which all ramain in the reasonable interval compared with GI quark model[23] ($T=0.18$ GeV^2, $\alpha_s=0.6$, $m=0.22$ GeV) and the string model[19]. One can see that quark model predictions for meson masses agree with those of the string model but are slightly better when J is relatively large as compared to the observed data. We stress that our fit for T is closer to the confining parameter of recent lattice simulation[27]: $T=(0.394)^2=0.155$.

6 Conclusions

We revisit the Regge-like spectra of the light mesons in the model of relativistic string and a semi-relativistic quark model. By employing the auxiliary field (AF) technique, an analytical mass formula for light mesons is proposed in the limit of the large quantum number($N=n+J$), from which a quasi-linear Regge-Chew-Frautschi(RCF) plot is obtained and the flavor dependence arises from the quark masses. When the predicted RCF mass squared plot is verified with the recent PDG data of the light mesons (π/b, ρ/a, η/h, f/ω), we find our predictions fit well with the observed data.

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夸克和弦模型中非奇异轻介子的类雷吉谱

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摘要： 本工作重新考察了相对论弦模型和准相对论夸克模型中轻介子的类雷吉谱。基于辅助场技术，提出了非奇异轻介子的一个解析质量公式，由此推导出了一个准线性的 Regge-Chew-Frautschi 轨迹关系，并将此关系用相应介子的实验数据 (PDG) 进行了检验。检验结果表明，和实验观测数据比较，夸克模型对介子质量的预言与弦模型的相符，但在角动量较大时，夸克模型的预言稍微优于弦模型。

关键词： 轻介子；雷吉轨迹；夸克模型，弦模型

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