# On the Origin of $21 / 2^{+}$Yrast Trap in ${ }^{93} \mathrm{Mo}$ 

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#### Abstract

Isomerism of the high－spin yrast $21 / 2^{+}$states of the $N=51$ isotones ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}$ has been investigated using the shell model calculations．It is found that the low－$j \pi p_{1 / 2}$ is responsible for the only yrast trap in ${ }^{93} \mathrm{Mo}$ ．In addition，the relatively smaller $10_{1}^{+}-12_{1}^{+}$level spacing in ${ }^{94} \mathrm{Mo}$ has been found by investigating the systematics of the $10_{1}^{+}-12_{1}^{+}$level structures in the $N=52$ isotones ${ }^{92} \mathrm{Zr},{ }^{94} \mathrm{Mo}$ and ${ }^{96} \mathrm{Ru}$ ． This result provides a supplementary argument to the origin of the $21 / 2^{+}$yrast trap in ${ }^{93} \mathrm{Mo}$ from the viewpoint of the similarity between the configurations of $10_{1}^{+}-12_{1}^{+}$states in ${ }^{94}$ Mo and those of $17 / 2_{1}^{+}-21 / 2_{1}^{+}$ states in ${ }^{93} \mathrm{Mo}$ ．


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## 1 Introduction

High－spin states in near－spherical nuclei can be constructed by aligned angular momentum of open shell nucleons until their full alignment．The fully－ aligned or terminal state may go down since the an－ gular momenta of valence proton and neutron are nearly parallel to each other and the associated proton－ neutron（p－n）interaction for particle－particle or hole－ hole coupling is strongly attractive．Given the inter－ actional strength，the p－n interaction is usually larger for larger $j$ value ${ }^{[1]}$ ．The low energy of the emitted $\gamma$ ray from the terminal state plays an important role in the onset of high－spin yrast isomers．To a certain degree，the terminal state intrudes into the low－spin region and a yrast trap（or spin gap）therefore oc－ curs，which can de－excite via the higher multipolarity $(\Delta I \geqslant 3) \gamma$ transition ${ }^{[2]}$ ．Much experimental effort has been devoted to searching for the high－spin yrast iso－ mers in $A \sim 100$ mass region ${ }^{[3-20]}$ ，where the high－$j$ $\pi g_{9 / 2}, \nu g_{9 / 2}, \nu g_{7 / 2}$ and $\nu h_{11 / 2}$ constitute the config－ urations of the terminal states．In these experiments， some yrast traps have been found（e．g．，the $21 / 2^{+}$in ${ }^{93} \mathrm{Mo}^{[4-6]}$ ，the $21^{+}$in ${ }^{94} \mathrm{Ag}^{[7]}$ ，the $23 / 2^{+}$in ${ }^{95} \mathrm{Ag}^{[8,9]}$ ，
the $16^{+}$in ${ }^{96} \mathrm{Cd}^{[10]}$ ，and the $25 / 2^{+}$in ${ }^{97} \mathrm{Cd}^{[11]}$ ），while some isomers don＇t give rise to the spin gaps（e．g．， the $21 / 2^{+}$in ${ }^{91} \mathrm{Zr}^{[12,13]}$ ，the $17 / 2^{-}$in ${ }^{91} \mathrm{Nb}^{[12,13]}$ ，the $14^{+}$in ${ }^{94} \mathrm{Pd}^{[14,15]}$ ，the $21 / 2^{+}$in ${ }^{95} \mathrm{Ru}^{[16]}$ ，the $15^{+}$in ${ }^{96} \mathrm{Ag}^{[10,17,18]}$ ，the $17^{-}$in ${ }^{98} \mathrm{Zr}^{[19]}$ ，and the $27 / 2^{-}$in $\left.{ }^{99} \mathrm{Mo}^{[20]}\right)$ ．The question about the yrast trap appear－ ing in some nuclei but missing in some other nuclei hasn＇t been solved satisfactorily．In this paper，we at－ tempt to research this puzzle from the aspect of $21 / 2_{1}^{+}$ states in ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}$ ．

It has been experimentally known that the $21 / 2_{1}^{+}$ states are high－spin isomers in the $N=51$ isotones ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}^{[21]}$ ．The main parts of level schemes of ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}$ are showed in Fig． 1.

In ${ }^{93} \mathrm{Mo}$ ，the $21 / 2_{1}^{+}$lies below the $15 / 2_{1}^{+}, 17 / 2_{1}^{+}$ and $19 / 2_{1}^{+}$and forms a yrast trap de－exciting to the $13 / 2_{1}^{+}$with $\Delta I=4$ different from the $21 / 2_{1}^{+} \rightarrow 17 / 2_{1}^{+}$ E2 isomeric transitions in ${ }^{91} \mathrm{Zr}$ and ${ }^{95} \mathrm{Ru}$ ．It has been indicated in Ref．［22］that the $21 / 2^{+}$yrast trap in ${ }^{93} \mathrm{Mo}$ certainly arises from the high－$j \pi g_{9 / 2}-\nu d_{5 / 2}$ interac－ tion．However，there is no such $21 / 2_{1}^{+}$yrast trap in the neighboring ${ }^{91} \mathrm{Zr}$ and ${ }^{95} \mathrm{Ru}^{[12,13,16]}$ although all the three $21 / 2_{1}^{+}$states are dominated by the termina－

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Fig． 1 The partial level schemes of $N=51$ isotones ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}^{[21,31]}$.
ting three－quasi－particle $\pi\left(g_{9 / 2}^{2}\right)_{8^{+}} \otimes \nu d_{5 / 2}$ configura－ tion ${ }^{[13,16,22]}$ ．The $\mathrm{P}+\mathrm{QQ}$－based interaction has suc－ cessfully described the the level structure of ${ }^{93,94,95} \mathrm{Mo}$ ， especially the characteristics of the $21 / 2^{+}$yrast trap in ${ }^{93} \mathrm{Mo}{ }^{[22,}{ }^{23]}$ ．Nevertheless，as for ${ }^{91} \mathrm{Zr}$ and ${ }^{95} \mathrm{Ru}$ ，the puzzle of the missing of $21 / 2^{+}$yrast trap has not been explained using the shell－model calculations．There－ fore，it is necessary to reinvestigate the reasons for the presence（missing）of the $21 / 2^{+}$trap in ${ }^{93} \mathrm{Mo}\left({ }^{91} \mathrm{Zr}\right.$ and ${ }^{95} \mathrm{Ru}$ ）by shell－model calculations with an uni－ fied effective interaction（e．g．，the Gloeckner－Serduke interaction $\left.{ }^{[24,25]}\right)$ ．

## 2 Model Space of the Present Shell Model Calculations

Since the Fermi levels of $\mathrm{Zr}-\mathrm{Ru}$ nuclei lie at the $\pi p_{1 / 2}$ or $\pi g_{9 / 2}$ and $\nu d_{5 / 2}$ orbits，the active pro－ ton and neutron orbits are therefore $\pi\left(p_{1 / 2}, g_{9 / 2}\right)$
and $\nu\left(d_{5 / 2}, s_{1 / 2}, d_{3 / 2}, g_{7 / 2}, h_{11 / 2}\right)$ ，respectively．The single－neutron energies of $d_{3 / 2}, g_{7 / 2}$ and $h_{11 / 2}$ are much higher with respect to the $d_{5 / 2}$ for the concerned states in Zr －Mo nuclei ${ }^{[26]}$ ．In addition， as regards the high－spin states up to $21 / 2_{1}^{+}$in ${ }^{95} \mathrm{Ru}$ ，their configurations are dominated by the coupling of $\left(p_{1 / 2}, g_{9 / 2}\right)$ proton and $\left(d_{5 / 2}, s_{1 / 2}\right)$ neu－ tron configurations ${ }^{[16,27,28]}$ ．Thus，we restrict pro－ tons and neutrons in the simple and clear subspaces： $\pi\left(p_{1 / 2}, g_{9 / 2}\right)$ and $\nu\left(d_{5 / 2}, s_{1 / 2}\right)$ ，which are suitable for the Gloeckner－Serduke interaction ${ }^{[24,25]}$ ．More－ over，the shell model calculations of ${ }^{91} \mathrm{Zr}$ and ${ }^{95} \mathrm{Ru}$ have been performed using the Gloeckner－Serduke interaction ${ }^{[13,16,27,28]}$ ，which was also taken as a part of two－body matrix elements in the shell－model calcu－ lation for ${ }^{93} \mathrm{Mo}^{[6,29]}$ ．

Therefore，we perform the shell－model cal－ culations for ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}$ with the code NuShellX ${ }^{[30]}$ using the Gloeckner－Serduke interaction ${ }^{[24,25]}$ with valence protons occupying the $p_{1 / 2}, g_{9 / 2}$ orbits and a valence neutron in the $d_{5 / 2}$ and $s_{1 / 2}$ orbits above the core ${ }^{88} \mathrm{Sr}$ core．The single－ particle and interactional energies of protons and neu－ trons in Gloeckner－Serduke interaction are presented in Table 1.

## 3 Results and Discussions

The calculated results for the $N=51$ and 52 iso－ tones of $\mathrm{Zr}, \mathrm{Mo}$ and Ru elements are presented in Fig．2，Tables 2 and 3．The calculated level energies

Table 1 Single－particle energy and two－body matrix elements（in unit of MeV ）of protons and neutrons in Gloeckner－ Serduke interaction ${ }^{[24,25]}$ ．

| Proton－proton | Value | Neutron－neutron | Value | Proton－neutron | Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\pi p_{1 / 2}$ single－particle energy | －7．125 | $\nu d_{5 / 2}$ single－particle energy | －6．338 | $\left\langle p_{1 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|p_{1 / 2} d_{5 / 2}\right\rangle_{J=2}$ | －0．585 |
| $\pi g_{9 / 2}$ single－particle energy | －6．247 | $\nu s_{1 / 2}$ single－particle energy | －5．506 | $\left\langle p_{1 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|p_{1 / 2} d_{5 / 2}\right\rangle_{J=3}$ | －0．358 |
| $\left\langle p_{1 / 2}^{2}\right\| V_{\mathrm{pp}}\left\|p_{1 / 2}^{2}\right\rangle_{J=0}$ | －0．542 | $\left\langle d_{5 / 2}^{2}\right\| V_{\mathrm{nn}}\left\|d_{5 / 2}^{2}\right\rangle_{J=0}$ | －0．908 | $\left\langle p_{1 / 2} s_{1 / 2}\right\| V_{\mathrm{pn}}\left\|p_{1 / 2} s_{1 / 2}\right\rangle_{J=0}$ | －0．143 |
| $\left\langle p_{1 / 2}^{2}\right\| V_{\mathrm{pp}}\left\|g_{9 / 2}^{2}\right\rangle_{J=0}$ | 0.853 | $\left\langle d_{5 / 2}^{2}\right\| V_{\mathrm{nn}}\left\|d_{5 / 2}^{2}\right\rangle_{J=2}$ | －0．384 | $\left\langle p_{1 / 2} s_{1 / 2}\right\| V_{\mathrm{pn}}\left\|p_{1 / 2} s_{1 / 2}\right\rangle_{J=1}$ | 0.143 |
| $\left\langle p_{1 / 2} g_{9 / 2}\right\| V_{\mathrm{pp}}\left\|p_{1 / 2} g_{9 / 2}\right\rangle_{J=4}$ | 0.714 | $\left\langle d_{5 / 2}^{2}\right\| V_{\mathrm{nn}}\left\|d_{5 / 2}^{2}\right\rangle_{J=4}$ | 0.146 | $\left\langle g_{9 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} d_{5 / 2}\right\rangle_{J=2}$ | －0．792 |
| $\left\langle p_{1 / 2} g_{9 / 2}\right\| V_{\mathrm{pp}}\left\|p_{1 / 2} g_{9 / 2}\right\rangle_{J=5}$ | 0.195 | $\left\langle d_{5 / 2}^{2}\right\| V_{\mathrm{nn}}\left\|d_{5 / 2} s_{1 / 2}\right\rangle_{J=2}$ | －0．291 | $\left\langle g_{9 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} d_{5 / 2}\right\rangle_{J=3}$ | －0．461 |
| $\left\langle g_{9 / 2}^{2}\right\| V_{\mathrm{pp}}\left\|g_{9 / 2}^{2}\right\rangle_{J=0}$ | －1．707 | $\left\langle d_{5 / 2}^{2}\right\| V_{\mathrm{nn}}\left\|s_{1 / 2}^{2}\right\rangle_{J=0}$ | －1．097 | $\left\langle g_{9 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} d_{5 / 2}\right\rangle_{J=4}$ | 0.182 |
| $\left\langle g_{9 / 2}^{2}\right\| V_{\mathrm{pp}}\left\|g_{9 / 2}^{2}\right\rangle_{J=2}$ | －0．613 | $\left\langle d_{5 / 2} s_{1 / 2}\right\| V_{\mathrm{nn}}\left\|d_{5 / 2} s_{1 / 2}\right\rangle_{J=2}$ | －0．106 | $\left\langle g_{9 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} d_{5 / 2}\right\rangle_{J=5}$ | －0．161 |
| $\left\langle g_{9 / 2}^{2}\right\| V_{\mathrm{pp}}\left\|g_{9 / 2}^{2}\right\rangle_{J=4}$ | 0.144 | $\left\langle d_{5 / 2} s_{1 / 2}\right\| V_{\mathrm{nn}}\left\|d_{5 / 2} s_{1 / 2}\right\rangle_{J=3}$ | －0．019 | $\left\langle g_{9 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} d_{5 / 2}\right\rangle_{J=6}$ | －0．079 |
| $\left\langle g_{9 / 2}^{2}\right\| V_{\mathrm{pp}}\left\|g_{9 / 2}^{2}\right\rangle_{J=6}$ | 0.450 | $\left\langle s_{1 / 2}^{2}\right\| V_{\mathrm{nn}}\left\|s_{1 / 2}^{2}\right\rangle_{J=0}$ | －0．598 | $\left\langle g_{9 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} d_{5 / 2}\right\rangle_{J=2}$ | －0．716 |
| $\left\langle g_{9 / 2}^{2}\right\| V_{\mathrm{pp}}\left\|g_{9 / 2}^{2}\right\rangle_{J=8}$ | 0.565 |  |  | $\left\langle g_{9 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} s_{1 / 2}\right\rangle_{J=4}$ | 0.550 |
|  |  |  |  | $\left\langle g_{9 / 2} d_{5 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} s_{1 / 2}\right\rangle_{J=5}$ | －0．515 |
|  |  |  |  | $\left\langle g_{9 / 2} s_{1 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} s_{1 / 2}\right\rangle_{J=4}$ | －0．644 |
|  |  |  |  | $\left\langle g_{9 / 2} s_{1 / 2}\right\| V_{\mathrm{pn}}\left\|g_{9 / 2} s_{1 / 2}\right\rangle_{J=5}$ | －0．555 |



Fig. 2 (color online) Comparison of the experimental and calculated excitation energies of (a) the yrast states up to $21 / 2_{1}^{+}$in ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}$, and (b) the yrast states up to $12_{1}^{+}$in ${ }^{92} \mathrm{Zr},{ }^{94} \mathrm{Mo}$ and ${ }^{96} \mathrm{Ru}$. Experimental data are taken from Ref. [21] except for the $15 / 2^{+}$and $19 / 2^{+}$states in ${ }^{91} \mathrm{Zr}$ from Ref. [31]. Some curves are shifted by adding $\Delta$ in MeV for clear illustration.

Table 2 Experimental and calculated reduced transition probabilities $B(E \lambda)$ in W.u. related to the decay of the isomeric $21 / 2_{1}^{+}$states in ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}$. The experimental values are taken from Ref. [21] and the effective charges are taken as $e_{\mathrm{p}}=2.1 e$ and $e_{\mathrm{n}}=1.0 e$ in the calculation.

| Nucleus | Transition | $\lambda$ | $B(E \lambda)_{\text {exp. }}$ | $B(E \lambda)_{\text {cal. }}$ |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{91} \mathrm{Zr}$ | $21 / 2_{1}^{+} \rightarrow$ | 2 | $4.3(7)$ | 3.66 |
|  | $17 / 2_{1}^{+}$ |  |  |  |
| ${ }^{93} \mathrm{Mo}$ | $21 / 2_{1}^{+} \rightarrow$ | 4 | $1.449(17)$ | 1.93 |
|  | $13 / 2_{1}^{+}$ |  |  |  |
| ${ }^{93} \mathrm{Mo}$ | $17 / 2_{1}^{+} \rightarrow$ | 2 | $4.48(23)$ | 4.05 |
|  | $13 / 2_{1}^{+}$ |  |  |  |
| ${ }^{95} \mathrm{Ru}$ | $21 / 2_{1}^{+} \rightarrow$ | 2 | $1.94(5)$ | 1.65 |
|  | $17 / 2_{1}^{+}$ |  |  |  |

Table 3 Main components of the wave functions of the partial yrast states in the $N=51$ and 52 even- $Z$ $\mathrm{Zr}-\mathrm{Ru}$ isotones calculated in the $\pi\left(p_{1 / 2}, g_{9 / 2}\right)$ and $\nu\left(d_{5 / 2}, s_{1 / 2}\right)$ model space.

| Nucleus | $I^{\pi}$ | Leading configuration | Partition $/ \%$ |
| :---: | :---: | :---: | :---: |
| ${ }^{91} \mathrm{Zr}$ | $21 / 2_{1}^{+}$ |  | 100 |
|  | $19 / 2_{1}^{+}$ |  | 100 |
|  | $17 / 2_{1}^{+}$ | $\pi g_{9 / 2}^{2} \otimes \nu d_{5 / 2}$ | 85.30 |

Table 3 (Continued)

| Nucleus | $I^{\pi}$ | Leading configuration | Partition/\% |
| :---: | :---: | :---: | :---: |
|  | $15 / 2_{1}^{+}$ |  | 63.68 |
|  | $13 / 2_{1}^{+}$ |  | 80.70 |
| ${ }^{93} \mathrm{Mo}$ | $21 / 2_{1}^{+}$ |  | 86.60 |
|  | $19 / 2_{1}^{+}$ | $\pi\left(p_{1 / 2}^{2} g_{9 / 2}^{2}\right) \otimes \nu d_{5 / 2}$ | 76.23 |
|  | $17 / 2_{1}^{+}$ |  |  |
|  | $15 / 2_{1}^{+}$ |  | 73.04 |
|  | $13 / 2_{1}^{+}$ |  | 76.58 |
| ${ }^{95} \mathrm{Ru}$ | $21 / 2_{1}^{+}$ |  | 88.35 |
|  | $19 / 2_{1}^{+}$ | $\pi\left(p_{1 / 2}^{2} g_{9 / 2}^{4}\right) \otimes \nu d_{5 / 2}$ | 97.23 |
|  | $17 / 2_{1}^{+}$ |  |  |
|  | $15 / 2_{1}^{+}$ |  | 65.08 |
|  | $13 / 2_{1}^{+}$ |  | 82.34 |
| ${ }^{92} \mathrm{Zr}$ | $12_{1}^{+}$ | $\pi g_{9 / 2}^{2} \otimes \nu d_{5 / 2}^{2}$ | 100 |
|  | $10_{1}^{+}$ |  | 51.33 |
| ${ }^{94} \mathrm{Mo}$ | $12_{1}^{+}$ | $\pi\left(p_{1 / 2}^{2} g_{9 / 2}^{2}\right) \otimes \nu d_{5 / 2}^{2}$ | 87.64 |
|  | $10_{1}^{+}$ | 74.44 |  |
| ${ }^{96} \mathrm{Ru}$ | $12_{1}^{+}$ | $\pi\left(p_{1 / 2}^{2} g_{9 / 2}^{4}\right) \otimes \nu d_{5 / 2}^{2}$ | 89.17 |
|  | $10_{1}^{+}$ |  | 80.43 |

and the isomeric $B(E \lambda)$ values are in good agreement with the experimental ones ${ }^{[21,31]}$ although the calculated $B\left(\mathrm{E} 4 ; 21 / 2^{+} \rightarrow 13 / 2^{+}\right)$value of ${ }^{93} \mathrm{Mo}$ is not the smallest value compared with other calculated $B(\mathrm{E} 2)$ ones in Table 2 , which may be contributed to the relatively large deviation of calculated reduced electric transition probabilities from the experimental values. Then we calculated the level energies in the case of the pure configuration (PC): $\pi g_{9 / 2}^{2} \otimes \nu d_{5 / 2}$ for ${ }^{91} \mathrm{Zr}$, $\pi p_{1 / 2}^{2} g_{9 / 2}^{2} \otimes \nu d_{5 / 2}$ for ${ }^{93} \mathrm{Mo}, \pi p_{1 / 2}^{2} g_{9 / 2}^{4} \otimes \nu d_{5 / 2}$ for ${ }^{95} \mathrm{Ru}$. After that, the configuration mixing with one neutron going down from $d_{5 / 2}$ into $s_{1 / 2}$ and two protons jumping from $p_{1 / 2}$ into $g_{9 / 2}$ is taken into account, respectively (hereinafter denoted by Mix1 and Mix2 respectively). The results are shown in Fig. 3 (a)~(c) for odd- $A$ nuclei.

Fig. 3 (a) shows that the PC calculations could produce $\Delta I=4$ yrast trap in ${ }^{91} \mathrm{Zr}$ and ${ }^{93} \mathrm{Mo}$. The main configurations of the fully-aligned $21 / 2_{1}^{+}$are both the coupling of two $g_{9 / 2}$ protons to one $d_{5 / 2}$ neutron. According to the re-coupling rule, these terminal state only contain the attractive $\left(\pi g_{9 / 2} \nu d_{5 / 2}\right)_{6^{+}, 7^{+}}$matrix elements. Compared with the level spacings between the $17 / 2_{1}^{+}$and $21 / 2_{1}^{+}$states in ${ }^{91} \mathrm{Zr}$ and ${ }^{93} \mathrm{Mo}$, the $21 / 2_{1}^{+}$state in ${ }^{95} \mathrm{Ru}$ with four $g_{9 / 2}$ protons becomes higher relative to the $17 / 2_{1}^{+}$in the figure. This is attributed to the additional $\left(\pi g_{9 / 2} \nu d_{5 / 2}\right)_{2^{+}, 3^{+}, 4^{+}, 5^{+}}$ taking part in the interaction, which might be repulsive on average relative to the case of the $\left(\pi g_{7 / 2}\right.$. $\left.\nu d_{5 / 2}\right)_{6+, 7^{+}}{ }^{[32]}$. Thus, the high- $j$ p-n interactions induce the yrast trap in ${ }^{91} \mathrm{Zr}$ and ${ }^{93} \mathrm{Mo}$ rather than ${ }^{95} \mathrm{Ru}$.


Fig． 3 （color online）（a）The excitation energies of the $13 / 2_{1}^{+}-21 / 2_{1}^{+}$from the PC calculations in the $N=51 \mathrm{Zr}-\mathrm{Ru}$ isotones，which are also compared with the results of（b）the Mix1 and（c）the Mix2 calculations，respectively． （d）The calculated excitation energies of the $13 / 2_{1}^{+}-21 / 2_{1}^{+}$in ${ }^{93} \mathrm{Mo}$ corresponding to the monopole interaction $V\left(\pi p_{1 / 2} \nu d_{5 / 2}\right)=0$ and -0.453 MeV ．See text for details．Some curves are shifted by adding $\Delta$ in MeV for clear illustration．

Two points can be seen from Fig． 3 （b）and（c）． On the one hand，the $M i x 1$ leads the lowering of the yrast states except for the $19 / 2_{1}^{+}$and $21 / 2_{1}^{+}$states； hence the the $17 / 2_{1}^{+}$levels go down．On the other hand，there is no obvious effect on the level spacings between the $17 / 2_{1}^{+}$and $21 / 2_{1}^{+}$states with the Mix2 calculation．We also checked the mixing of the unper－ turbed $17 / 2_{1}^{+}$states with $\pi g_{9 / 2}^{4} \otimes \nu s_{1 / 2}$ configuration and found that it takes on the similar but smaller effect on configuration interaction as the case of $\pi g_{9 / 2}^{4} \otimes \nu d_{5 / 2}$ configuration．Thus，we can give a conclusion that the configuration mixing doesn＇t cause the inversion be－ tween the $17 / 2_{1}^{+}$and $21 / 2_{1}^{+}$levels in ${ }^{91} \mathrm{Zr},{ }^{93} \mathrm{Mo}$ and ${ }^{95} \mathrm{Ru}$ ．

We noted that the Mix1 calculation still yields the order inversion between $17 / 2_{1}^{+}$and $21 / 2_{1}^{+}$in ${ }^{91} \mathrm{Zr}$ though the energy difference is very small $(\sim 10 \mathrm{keV})$ ． If we neglect the discrepancy and assume the calcula－ tion giving rise to no inversion for ${ }^{91} \mathrm{Zr}$ ，it would spring up a puzzle about the presence of yrast trap in ${ }^{93} \mathrm{Mo}$ ， which after all has the similar configuration as the case of the isomer in ${ }^{91} \mathrm{Zr}$ ．

In order to reveal this mystery，we carefully
checked the wave functions of $17 / 2_{1}^{+}$and $21 / 2_{1}^{+}$states in ${ }^{93} \mathrm{Mo}$ ．When moving from $17 / 2_{1}^{+}$to $21 / 2_{1}^{+}$，the am－ plitude of unperturbed $\pi\left(p_{1 / 2}^{2} g_{9 / 2}^{2}\right) \otimes \nu d_{5 / 2}$ grows up from $77.96 \%$ to $86.60 \%$ ，although the growth seems small and the dominance of the configurations is not changed．As mentioned above，the configuration in－ teraction does not lead to the inverted levels．One naturally expects that the yrast trap arises from the amplitude variation．If the $p_{1 / 2}-d_{5 / 2} \mathrm{p}$－n interaction is attractive，the additional $8.64 \%$ amplitude of $21 / 2_{1}^{+}$ relative to that of $17 / 2_{1}^{+}$may result in the lowering of the $21 / 2_{1}^{+}$．Indeed，the $\pi p_{1 / 2}-\nu d_{5 / 2}$ interaction is strongly attractive．Taking into account the p－n in－ teraction diagonal interaction matrix elements of -585 keV and -358 keV with the angular momentums of $2^{-}$ and $3^{-}$respectively from the coupling of $\pi p_{1 / 2}$ and $\nu d_{5 / 2}$ ，the monopole interaction $V=-453 \mathrm{keV}$ can therefore be obtained ${ }^{[25]}$ ．This monopole interaction of course can reproduce the yrast trap in ${ }^{93} \mathrm{Mo}$（see Fig．3（d））．As the monopole interaction drops to zero， the inversion is completely washed out．The $17 / 2_{1}^{+}$ and $21 / 2_{1}^{+}$states in ${ }^{93} \mathrm{Mo}$ differ from those in ${ }^{91} \mathrm{Zr}$ by two $\pi p_{1 / 2}$ in pair．The cancellation of $\pi p_{1 / 2}-\nu d_{5 / 2}$ in－
teraction in ${ }^{91} \mathrm{Zr}$ can reasonably explain the missing of yrast trap. In Ref. [22], which successfully described the $21 / 2^{+}$isomer in ${ }^{93} \mathrm{Mo}$, the wave functions of $17 / 2^{+}$ and $21 / 2^{+}$states are dominated by $\pi\left(g_{9 / 2}^{2}\right)_{6^{+}} \otimes \nu d_{5 / 2}$ and $\pi\left(g_{9 / 2}^{2}\right)_{8}+\otimes \nu d_{5 / 2}$ respectively corresponding to the $\pi\left(p_{1 / 2}^{2} g_{9 / 2}^{2}\right) \otimes \nu d_{5 / 2}$ in our work. However, what we focus on is the change of the amplitudes of the leading configurations from $17 / 2^{+}$to $21 / 2^{+}$states, which leads us to access the ruled territory of $p_{1 / 2}-d_{5 / 2} p-n$ interaction and reveal the puzzle about the presence of $21 / 2^{+}$yrast trap in ${ }^{93}$ Mo but missing of that in ${ }^{91} \mathrm{Zr}$.

It can be seen from Table 3 that the $10_{1}^{+}$and fullyaligned $12_{1}^{+}$states in ${ }^{92} \mathrm{Zr}$ and ${ }^{96} \mathrm{Ru}$ have the dominant four quasi-particle configuration $\pi g_{9 / 2}^{2} \otimes \nu d_{5 / 2}^{2}$. Removing a $d_{5 / 2}$ neutron from them leads to the $17 / 2_{1}^{+}$and $21 / 2_{1}^{+}$states in $N=51$ isotones. Thus, the $12_{1}^{+}-10_{1}^{+}$ level spacing in ${ }^{94} \mathrm{Mo}(294 \mathrm{keV})$ is smaller than that in ${ }^{92} \mathrm{Zr}(651 \mathrm{keV})$ and ${ }^{96} \mathrm{Ru}(601 \mathrm{keV}){ }^{[21,23,31,33]}$ can be understood in quality by the analogy to the systematic trends of the $21 / 2_{1}^{+}-17 / 2_{1}^{+}$in ${ }^{91} \mathrm{Zr}(20.2 \mathrm{keV})^{[31]}$, ${ }^{93} \mathrm{Mo}(-4.85 \mathrm{keV}){ }^{[34]}$ and ${ }^{95} \mathrm{Ru}(254.7 \mathrm{keV}){ }^{[27]}$. This provides a supplementary argument to the presence of the yrast trap in ${ }^{93} \mathrm{Mo}$.

## 4 Summary

In summary, spherical shell model calculations have been performed using the code NuShellX with the Gloeckner-Serduke interaction in the $\pi\left(p_{1 / 2}, g_{9 / 2}\right)$ and $\nu\left(d_{5 / 2}, s_{1 / 2}\right)$ model space. We found that the $\pi g_{9 / 2} \nu d_{5 / 2}$ p-n interaction induce the yrast trap in ${ }^{91} \mathrm{Zr}$ and ${ }^{93} \mathrm{Mo}$ rather than ${ }^{95} \mathrm{Ru}$, while the configuration mixing trends to counteract it. It is the $\pi p_{1 / 2} \nu d_{5 / 2}$ p-n interaction that causes the formation of the yrast trap in ${ }^{93} \mathrm{Mo}$. The smaller $10_{1}^{+}-12_{1}^{+}$level spacing in ${ }^{94}$ Mo provides a supplementary argument to the presence of the yrast trap in ${ }^{93} \mathrm{Mo}$.

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## 关于 ${ }^{93} \mathrm{Mo}$ 中 $21 / 2^{+}$晕阱的来源

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摘要：本研究通过壳模型计算研究了 $N=51$ 的同中子素 ${ }^{91} \mathrm{Zr}, ~{ }^{93} \mathrm{Mo}$ 和 ${ }^{95} \mathrm{Ru}$ 中高自旋晕态 $21 / 2^{+}$的同核异能态现象。计算发现，低角动量的 $p_{1 / 2}$ 轨道上的质子是仅在 ${ }^{93} \mathrm{Mo}$ 中存在 $21 / 2^{+}$晕阱的主要原因。同时，本工作还研究了 $N=52$ 的同中子素 ${ }^{92} \mathrm{Zr}, ~{ }^{94} \mathrm{Mo}$ 和 ${ }^{96} \mathrm{Ru}$ 中 $10_{1}^{+}-12_{1}^{+}$能级结构的系统性，发现 ${ }^{94} \mathrm{Mo}$ 中的 $10_{1}^{+}-12_{1}^{+}$能级间隙相对最小，考虑到与 ${ }^{93} \mathrm{Mo}$ 的 $17 / 2_{1}^{+}-21 / 2_{1}^{+}$能级相似的组态，这一结果为 ${ }^{93} \mathrm{Mo}$ 中出现 $21 / 2^{+}$晕阱提供了补充性的论证。
关键词：同核异能态；晕阱；${ }^{93} \mathrm{Mo}$


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