Puzzle of the Λ_c Spectrum

LÜ Qifang[1,2], DONG Yubing[2,3], LIU Xiang[4,5,†], MATSUKI Takayuki[6,7]

(1. Department of Physics, Hunan Normal University, Changsha 410081, China;

2. Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China;

3. Theoretical Physics Center for Science Facilities (TPCSF), Chinese Academy of Sciences, Beijing 100049, China;

4. School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China;

5. Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China;

6. Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan;

7. Theoretical Research Division, Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan)

Abstract：There is a puzzle in the Λ_c^+ family, *i.e.*, one member with $J^P = 3/2^+$ is missing in a $L = 2$ multiplet which the heavy quark effective theory predicts, and J^P's of $\Lambda_c(2765)^+$ and $\Lambda_c(2940)^+$ are unknown. Using a light diquark picture to calculate baryon masses, we study possible assignments of two Λ_c's with unknown J^P and the missing Λ_c^+ with $3/2^+$ for $L = 2$, and we find the most probable possibility that the peak corresponding to $\Lambda_c(2880)^+$ actually includes a missing member with spin $3/2^+$ for $L = 2$ and that quantum numbers of $\Lambda_c(2765)^+$ and $\Lambda_c(2940)^+$ are $2S(1/2^+)$ and $2P(1/2^-)$, respectively.

Key words: Λ_c family; relativized quark model; diquark

CLC number: O572.24$^+$3   **Document code**: A   **DOI**: 10.11804/NuclPhysRev.35.01.001

1 Introduction

In the former paper[1], we have pointed out that careful observation of the experimental spectra of heavy-light mesons tells us that heavy-light mesons with the same angular momentum L are almost degenerate and that mass differences within a heavy quark spin doublet and between doublets with the same L are very small compared with a mass gap between different multiplets with different L, which is nearly equal to the value of the $\Lambda_{\rm QCD} \sim 300$ MeV. In Conclusions and Discussion of Ref. [1], we have also suggested that Λ_c^+ baryons may have properties similar to heavy-light mesons. There have been a couple of papers which pursue the similar idea for light and vector mesons by Afonin and his collaborators[2, 3], on which we do not discuss in this paper.

According to PDG[4], there are six Λ_c^+ baryons observed by experiments, which are $\Lambda_c(2286)^+$, $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Lambda_c(2765)^+$ (or $\Sigma_c(2765)^+$), $\Lambda_c(2880)^+$, and $\Lambda_c(2940)^+$. Among these, the following heavy quark spin multiplets are identified;

$\Lambda_c(2286)$ with $J^P = 1/2^+$ for $L = 0$, $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ with $1/2^+$ and $3/2^+$ for $L = 1$, respectively, and $\Lambda_c(2880)^+$ with $5/2^+$ for $L = 2$. There is one member missing in the spin multiplet for $L = 2$ which has spin $3/2^+$. Other than this missing particle, mass differences in the same multiplet, *i.e.*, with the same L, are very small and gaps between the average masses of spin multiplets are all ~ 300 MeV, which obeys the rule proposed in Ref. [1]. Although these successful assignments, there still remains a puzzle in the Λ_c^+ baryons, where the missing member for $L = 2$ is and what quantum numbers J^P are for $\Lambda_c(2765)^+$ and $\Lambda_c(2940)^+$. If we regard $\Lambda_c(2940)^+$ as a missing member of $L = 2$, then this state must have spin-parity $3/2^+$, which contradicts a common understanding that a state with smaller spin $3/2$ appears lower in mass than a state with larger spin $5/2$. In addition, the strong and radiative decays of $\Lambda_c(2940)$ in a D^*N molecular scenario have been analyzed in Refs. [5, 6] which are against spin $3/2$. Additionally, there is also another theoretical study of the production of the $\Lambda_c(2940)^+$ in $\pi^- {\rm p} \to {\rm D}^- {\rm D}^0 {\rm p}$ reaction[7], where the final ${\rm D}^0{\rm p}$ re-

sults from the $\Lambda_c(2940)^+$ state with spin-parity $1/2^+$ or $1/2^-$.

There is a pioneering work[8] which calculates baryon masses directly extending the method of Godfrey and Isgur[9]. However, since this method is complicated, in this article regarding a baryon as a heavy quark-light diquark system (Q{qq}) like in Ref. [10] and calculating its mass, we show that the above observation on Λ_c holds, predict J^P and spin assignments of $\Lambda_c(2765)^+$ and $\Lambda_c(2940)^+$, and propose a solution to a puzzle where the missing member for $L = 2$ is.

We adopt the method provided in Ref. [10] to calculate baryon masses whose prescription is given by: 1) First we calculate diquark masses assuming the relativised quark model proposed by Godfrey and Isgur (GI)[9]. 2) Next, having a diquark mass calculated and regarding two light quarks as $3^* \in 3{\times}3$, we again apply the GI model to a heavy quark-light diquark system to obtain a baryon mass.

This way of calculation places very tight conditions on parameters so that it is very difficult to reproduce physical masses as one can imagine. Hence, we also refer to the values of baryon masses where diquark masses are parameters to fit with experimental data[11], and also refer to the values given in Refs. [8, 10, 12].

2　Relativized Quark Model and Diquark Masses

To calculate baryon masses, we adopt interactions proposed by the relativized GI model whose Hamiltonian between quark and antiquark can be expressed as

$$\tilde{H} = H_0 + \tilde{V}(\boldsymbol{p}, \boldsymbol{r}) , \tag{1}$$

where

$$H_0 = (p^2 + m_1^2)^{1/2} + (p^2 + m_2^2)^{1/2} , \tag{2}$$

$$\tilde{V}(\boldsymbol{p}, \boldsymbol{r}) = \tilde{H}_{12}^{\text{conf}} + \tilde{H}_{12}^{\text{cont}} + \tilde{H}_{12}^{\text{ten}} + \tilde{H}_{12}^{\text{so}}. \tag{3}$$

Here $\tilde{H}_{12}^{\text{conf}}$ includes the spin-independent linear confinement and Coulomb-like interaction. $\tilde{H}_{12}^{\text{cont}}$, $\tilde{H}_{12}^{\text{ten}}$, and $\tilde{H}_{12}^{\text{so}}$ are the color contact, color tensor interactions, and spin-orbit term, respectively. Subindices 1 and 2 denote quark ($\boldsymbol{3_c}$) and antiquark ($\boldsymbol{3_c^*}$), respectively. The symbol "∼" on top of the operator $\tilde{H}$ means that we operate the relativized procedure on H, by which relativistic effects are taken into account. The explicit forms of those interactions and the details of the relativization procedure can be found in Refs. [8, 9] for mesons and baryons, respectively.

To solve Eq. (1), we need values of parameters which are given in Table 1 provided by Refs. [8, 9].

Here C_{qq} is taken to be the same as $C_{\text{q}\bar{\text{q}}}$ in Ref. [9] up to the factor, an inner product of color matrices $\langle \boldsymbol{F}_1 \cdot \boldsymbol{F}_2 \rangle$, with $V_{\text{string}}(\boldsymbol{r}) = C_{\text{qq}}(\text{or } C_{\text{q}\bar{\text{q}}}) + br$. $\tilde{V}$ includes Gell-Man matrices whose expectation vlaues, $\langle \boldsymbol{F}_1 \cdot \boldsymbol{F}_2 \rangle$, are 4/3 for q$\bar{\text{q}}$ and $-2/3$ for a diquark qq. In Table 1, GI means parameters taken from Ref. [9] and CI from Ref. [8] which we adopt in this work to calculate diquark masses as well as heavy quark-diquark, i.e., charmed baryon masses. In this work, $C_{\text{q}\bar{\text{q}}}$ is used for C_{Qdi} where Q is a heavy quark and "di" expresses a diquark. The word "same" in the CI column in Table 1 means the same value as a GI parameter.

Table 1　Values of parameters used in this work. GI means parameters taken from Ref. [9] and CI from Ref. [8].

	GI	CI (This work)
$\frac{1}{2}(m_{\text{u}}+m_{\text{d}})$/MeV	220	same
m_{s}/MeV	419	same
m_{c}/MeV	1628	same
α_{s}	0.60	same
Λ_{QCD}/MeV	200	same
$C_{\text{q}\bar{\text{q}}}$/MeV	$-\frac{4}{3}(-253)$	same
C_{qq}/MeV	N/A	$-\frac{2}{3}(-253)$
σ_0/GeV	1.80	same
s	1.55	same
b/GeV^2	0.18	0.15
$\frac{1}{2}+\epsilon_{\text{cont}}$	$\frac{1}{2}-0.168$	same
$\frac{1}{2}+\epsilon_{\text{tens}}$	$\frac{1}{2}+0.025$	$\frac{1}{2}-0.168$
$\frac{1}{2}+\epsilon_{\text{so}(v)}$	$\frac{1}{2}+0.055$	$\frac{1}{2}$
$\frac{1}{2}+\epsilon_{\text{so}(s)}$	$\frac{1}{2}+0.055$	$\frac{1}{2}+0.30$
$\frac{1}{2}+\epsilon_{\text{Coul}}$	$\frac{1}{2}$	same

The calculated diquark masses by using both parameter sets of GI and CI are listed in Table 2. Although Ref. [8] includes the three body interaction, we neglect it to simplify the calculation.

Table 2　Masses of scalar and axial vector diquarks. S and A denote scalar and axial vector diquarks, respectively. The braces and brackets correspond to symmetric and antisymmetric quark contents in flavor, respectively. The units are in MeV.

Quark content	Diquark type	Mass (GI)	Mass(CI)
[u, d]	S	691	642
{u, d}	A	840	779
[u, s]	S	886	838
{u, s}	A	992	934

3　Baryon masses

After obtaining diquark masses in Table 2, we calculate baryon masses using Eq. (1) with both parameter sets of GI and CI. Although the mass values with

the CI parameter set is better than those of GI, we list both results in Table 3 as "Prediction (CI/GI)" together with experimental data and other results given by Refs. [8, 10–12]. Among these results, Refs. [10, 11] use heavy quark-diquark picture and Ref. [11] uses the semi-relativistic quark potential model which should be compared with our results. Since Ref. [11] treats diquark masses as parameters, they obtain better fit with experimental data than ours. Our self-consistent calculation of the Λ_c baryon masses gives a rather higher values compared with other models. Even though our calculated value 2 930 MeV is very close to $\Lambda_c(2940)$, it is natural to consider that our values 2 930 and 2 919

MeV should form a multiplet for $L = 2$. In this case, we find that mass difference betweeen members of a $L = 2$ multiplet is within $\sim$10 MeV including other models. Accordingly, we can see the similar tendency for all the models:

(1) $\Lambda_c(2765)^+$ is identified as a $|\,2S,1/2^+\rangle$ state.

(2) $\Lambda_c(2940)^+$ is identified as a $|\,2P,1/2^-\rangle$ state.

(3) The observed peak of the $\Lambda_c(2880)^+$ assigned as $|\,1D,5/2^+\rangle$ actually includes a missing state $|\,1D,3/2^+\rangle$ because their predicted masses listed in Table 3 are so close to each other, within $\sim$10 MeV, which could not be distinguished by experiments.

Table 3　Predicted masses for the Λ_c^+ state of ours and other approaches in Refs. [8, 10–12] compared to experimental data[4] (in MeV). We adopt masses generated by the CI parameter set in this work.

States	Λ_c^+ baryons						
	PDG[4]	Prediction (CI)	Prediction (GI)	Ref. [11]	Ref. [10]	Ref. [8]	Ref. [12]
$|\,1S,1/2^+\rangle$	2 286.9	2177	2267	2286	2286	2265	2268
$|\,2S,1/2^+\rangle$	2 766.6	2749	2891	2772	2769	2775	2791
$|\,3S,1/2^+\rangle$		3160	3345	3116	3130	3170	
$|\,1P,1/2^-\rangle$	2 592.3	2603	2736	2614	2598	2630	2625
$|\,1P,3/2^-\rangle$	2 628.1	2619	2739	2639	2627	2640	2636
$|\,1D,3/2^+\rangle$		2930	3095	2843	2874	2910	2887
$|\,1D,5/2^+\rangle$	2 881.5	2919	3065	2851	2880	2910	2887
$|\,2P,1/2^-\rangle$	2 939.3	3030	3204	2980	2983	3030	2816
$|\,2P,3/2^-\rangle$		3038	3203	3004	3005	3035	2830

Items 2 and 3 have been already pointed out by the paper of Ref. [11]. As for Item 3, experimental errors of the mass and width for $\Lambda_c(2880)^+$ are so small, 2881.50 ± 0.35 and 5.8 ± 1.1 given in Refs. [4, 13, 14], respectively, that one cannot imagine that there is a missing particle hidden in the same peak at $\Lambda_c(2880)^+$. However, there are theoretical uncertainties as one can see from Table 3. Refs. [8, 12] give the same mass values for $|\,1D,5/2^+\rangle$ and $|\,1D,3/2^+\rangle$, and Refs. [10, 11] including our study give masses within $\sim$10 MeV so that considering theoretical errors, both states are most probably in one peak or it is very difficult to separate two states from the peak at 2880 MeV.

4　Conclusions and discussion

After many XYZ particles have been discovered, people have paid attention to these particles to solve a question how they can be explained, either molecular state, or tetra-quark state or just a kinematical effect[15–17]. Now settling most of the XYZ particles, people are now paying most of their energy to study heavy baryons, the cases that one or two or three quarks are heavy. To attack this problem is a

bit hard compared with the XYZ particles because baryons consist of three quarks and it is very hard to solve a three-body problem. A diquark picture makes the problem easier to calculate their mass spectrum as well as their decay behaviors with a help of the $^3P_0$ model as the paper (Ref. [18]). Also see the paper (Ref. [11]) how to apply the method proposed by Ref. [18] to baryons.

After starting from the simple observation that many of the heavy-light mesons have degenerate masses within a heavy quark spin multiplet, we could extend the idea to baryons. The first example was the Λ_c baryons. However, there is the puzzle in this spectrum that there is a missing member in a $L = 2$ heavy-quark spin multiplet.

In this article, we have studied the Λ_c^+ spectrum. To do so, we have calculated baryon masses taking a heavy quark-diquark picture for baryons and have compared with other theoretical as well as experimental values. To take a diquark picture for two light quarks, we have tried to be self-consistent, *i.e.*, we have calculated diquark masses at first using the GI model with the CI parameter set, and using those diquark masses, we have computed baryon masses. After observing the obtained values together with former theo-

retical values, we have concluded that $\Lambda_c(2765)^+$ and $\Lambda_c(2940)^+$ are identified as $|2S, 1/2^+\rangle$ and $|2P, 1/2^-\rangle$ states, respectively and that a missing member of the $L=2$ heavy-quark spin multiplet is hidden in the peak around $\Lambda_c(2880)^+$. To elaborate our argument. We are recalculating baryon masses including three-body interactions which is now in progress.

Because our prediction on a missing member of the $L=2$ heavy quark multiplet depends on accuracy of experiments, future careful measurements on the Λ_c^+ spectrum by LHCb and the forthcoming BelleII are waited for to test our prediction.

References:

[1] MATSUKI T, LÜ Q F, DONG Y B, *et al*. Phys Lett B, 2016, **758**: 274.

[2] AFONIN S S. Phys Rev C, 2007, **76**, 015202.

[3] AFONIN S S, PUSENKOV I V. Mod Phys Lett A, 2014, **29**: 1450193.

[4] OLIVE K A[Particle Data Group Collaboration]. Chin Phys C, 2014, **38**: 090001.

[5] DONG Y B, FAESSLER A, GUTSCHE T, *et al*. Phys Rev D, 2010, **81**: 014006.

[6] DONG Y B, FAESSLER A, GUTSCHE T,*et al*. Phys Rev D, 2010, **82**: 034035.

[7] XIE J J, DONG Y B, CAO X. Phys Rev D, 2015, **92**: 034029.

[8] CAPSTICK S, ISGUR N. Phys Rev D, 1986, **34**: 2809.

[9] GODFREY S, ISGUR N. Phys Rev D, 1985, **32**: 189.

[10] EBERT D, FAUSTOV R N, GALKIN V O. Phys Rev D, 2011, **84**: 014025.

[11] CHEN B, WEI K W, LIU X, MATSUKI T. arXiv:1609.07967 [hep-ph].

[12] ROBERTS W, PERVIN M. Int J Mod Phys A, 2008, **23**: 2817.

[13] ABE K[Belle Collaboration]. Phys Rev Lett, 2007, **98**: 262001.

[14] AUBERT B[BaBar Collaboration]. Phys Rev Lett, 2007, **98**: 012001.

[15] LIU X. Chin Sci Bull, 2014, **59**: 3815.

[16] CHEN H X, CHEN W, LIU X, *et al*. Phys Rept, 2016, **639**: 1.

[17] LEBED R F, MITCHELL R E, SWANSON E S. arXiv:1610.04528 [hep-ph].

[18] EICHTEN E J, HILL C T, QUIGG C. Phys Rev Lett, 1993, **71**: 4116.

Λ_c 重子谱疑难

吕齐放[1,2]，董宇兵[2,3]，刘 翔[4,5,†]，MATSUKI Takayuki[6,7]

(1. 湖南师范大学物理系，长沙 410081;
2. 中国科学院高能物理研究所，北京 100049;
3. 中国科学院大科学装置理论物理研究中心，北京 100049;
4. 兰州大学物理科学与技术学院，兰州 730000;
5. 兰州大学&中国科学院近代物理研究所，强子与CSR物理研究中心，兰州 730000;
6. Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan;
7. Theoretical Research Division, Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan)

摘要： Λ_c^+ 重子家族存在一个疑难，也就是重夸克有效理论预言的轨道角动量为 $L=2$ 的多重态中 $J^P = 3/2^+$ 的态在实验上是缺失的，而且已发现的 $\Lambda_c(2765)^+$ 和 $\Lambda_c(2940)^+$ 的 J^P 量子数仍然未知。在本工作中，我们使用双夸克图像计算了重子的质量，研究了 $\Lambda_c(2765)^+$ 和 $\Lambda_c(2940)^+$ 的可能归类以及仍缺失的具有 $L=2$ 和 $J^P = 3/2^+$ 量子数的 Λ_c^+ 态，进而指出 $\Lambda_c(2880)^+$ 的峰结构中实际上已经包含了 $L=2$、$J^P = 3/2^+$ 这一缺失态，并建议 $\Lambda_c(2765)^+$ 和 $\Lambda_c(2940)^+$ 的量子数应分别为 $2S(1/2^+)$ 和 $2P(1/2^-)$。

关键词： Λ_c 家族；相对论夸克模型；对夸克