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Evidence for Rotational to Vibrational Evolution Along the Yrast Line in the Odd-A Rare-earth Nuclei

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Abstract: The phase transition of nuclei with increasing angular momentum (or spin) and excitation energy is one of the most fundamental topics of nuclear structure research. The odd-N nuclei with $A \approx 160$ are widely considered belonging to the well-deformed region, and their excitation spectra are energetically favored to exhibit the rotational characteristics. In this work, however, the evidence suggesting that the nuclei changes from rotation to vibration along the yrast lines as a function of spin was found. The simple method, named as E-Gamma Over Spin (E-GOS) curves, would be used to discern the evolution from rotational to vibrational structure in nuclei for various spin ranges. Meanwhile, in order to understand the band structure properties of nuclei, theoretical calculations have been performed for the yrast bands of the odd-A rare-earth nuclei within the framework of the total routhian surface (TRS) model. The TRS plots predict that the ^{165}Yb and ^{157}Dy isotopes have large quadrupole shapes at low spin states. At higher rotational frequency ($\hbar\omega$ >0.50 MeV), a clear reduction of the quadrupole deformation is indicated by the present results, and the isotopes become rigid in the γ deformation.

Key words: phase transition; E-GOS curve; total Routhian surfaces

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1 Introduction

The phase transition as a function of spin is one of the most significant topics in nuclear structure research. This transition is intimately related to the mechanisms how atomic nuclei generate angular momentum. Recently, an abundance of observed phenomena connected with different collective band structures are well established by means of in-beam γ–ray spectroscopy[1–3]. The different characteristics of the excitation spectra manifest their angular momentum generation in different ways. For a nonspherical nucleus, the excited states can be formed by collective vibration or rotation motions, and a subtle rearrangement of only a few nucleons among the orbitals near the Fermi surface can result in completely different collective modes[4]. A simple method, so called the E-Gamma Over Spin (E-GOS) curve proposed by Regan *et al.*[5], has been used to distinguish directly between collective vibration and rotation excitations. Based on this prescription, a clear structure evolution from vibration to rotation with spin increasing was confirmed in the yrast cascades of the even-even nuclei in $A \sim 110$ region[5–6].

As is well known, nuclei in the mass region $150 < A < 190$ are considered belonging to the well-deformed region, and then one may infer that the yrast band consequently exhibits a rotational structure. Some recent findings[7], however, have given the evidences for a particularly interesting phenomenon for even-even nuclei in this mass region, *i.e.*, the evolution from rotation to vibration with the spin increasing, observed

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in the excitation spectra. Generally, the excited states of the odd-A nuclide can be considered as a quasi-particle coupling weakly to the respective core excitations. Therefore, one may have a good chance to observe the similar structure transition as their neighboring even-even nuclei in the area of 150< A <190. In this work, the collective modes of an odd-A nucleus as a function of spin were investigated, and the mechanism of this phase transition was discussed in the framework of the cranked shell model.

2 The *E*-GOS curve method

The concept of the E-GOS prescription has been applied to discern the structure evolution from vibration to rotation in nuclei as increasing spin[5]. In this method, the ratio of $E_\gamma(I\to I\text{-}2)/I$ can provide an effective way to distinguish the axially symmetric rotational and harmonic vibrational modes[5]. For the harmonic vibrator, this ratio decreases towards zero ultimately, whereas for the axially symmetric rotor it turns to a constant value of $4[\hbar^2/2J]$ with spin increasing, where J is the static moment of inertia. As an example, the E-GOS curves for a perfect harmonic vibrator and axially symmetric rotor with assuming the first excitations of 500 and 100 keV, respectively, are shown in Fig. 1(a). This prescription can be used as a quite good signature to discern the transition from vibrational to rotational structure in nuclei as a function of spin. Therefore, we expect that the ratio $E_\gamma(I\to I\text{-}2)/I$ can also distinguish the transition from rotational to vibrational motions. For the reader's convenience, the typical corresponding E-GOS curve for the axially rotational to vibrational structure evolution along the yrast line is also presented in Fig. 1(b), which is just the same as the Fig. 1(c) in Ref. [7]. In odd-A systems, however, the effect of the bandhead spin should be taken into account. Then, the E-GOS prescriptions can be addressed by substituting the spin (I) by a normalized spin minus the bandhead spin projection on the axis of symmetry, K, such that $I\to(I-K)$. For good rotors, the E-GOS prescription for odd-A systems then becomes[8]

$$R(I)=\frac{E_\gamma}{I}\to\frac{\hbar}{2J}\frac{(4I-2)}{I}\to\frac{\hbar^2}{2I}\frac{[4(I-K)]-2}{(I-K)}. \qquad (1)$$

For convenience, the spin I in expression (1) is replaced by $I-K$. The above formula can be written as

$$R(I-K)=\frac{E_\gamma-\left(4K\frac{\hbar^2}{2J}\right)}{I-K}=\frac{E_\gamma-KR_{K+2}}{I-K}. \qquad (2)$$

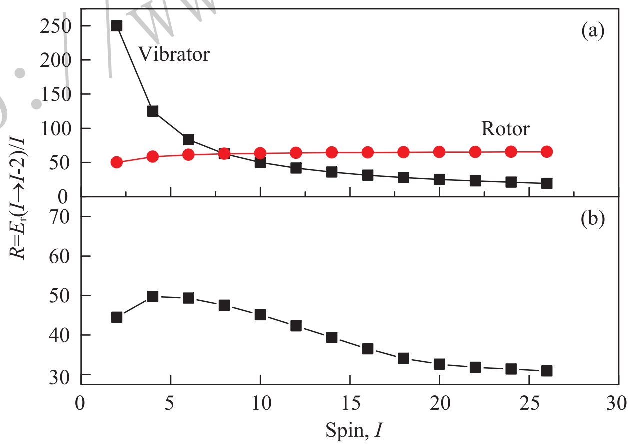

Fig. 1 (color online) (a) E-GOS curves for a perfect harmonic vibrator and axially symmetric rotor. (b) The characteristic of E-GOS plot for the axially rotational to vibrational shape transition with spin increasing.

Fig. 2 shows the E-GOS plots for the odd-N isotopes in the mass region around A=160. The data are taken from Refs. [9–27]. By comparing with the E-GOS curve characteristics presented in Fig. 1(b), the E-GOS plots of ^{155}Gd,$^{155-161}$Dy and $^{157-167}$Er shown in Fig. 2 present a clear evolution from rotational to vibrational excitations along the yrast line with increasing spin. Normally the Ytterbium and Hafnium isotopes given in Fig. 2 are considered belonging to the well-deformed region, and the good rotational energy spectra should be observed in these nuclei, and then every nucleus consequently exhibits a band structure with the yrast rotational band. So it is interesting to see that the yrast bands of $^{165-171}$Yb and ^{171}Hf have the vibrational characteristic at the lower-spin region, whereas it has a rotational pattern with higher spin. Particularly, it should be mentioned that the results for the highest spins (last four points) of ^{167}Yb, ^{171}Hf and (last three points) of ^{165}Yb in Fig. 2 point to an interesting aspect, the E-GOS curve in this region changes into the hyperbola expected for a vibrator. This suggests that above spin 35 there is again a change to

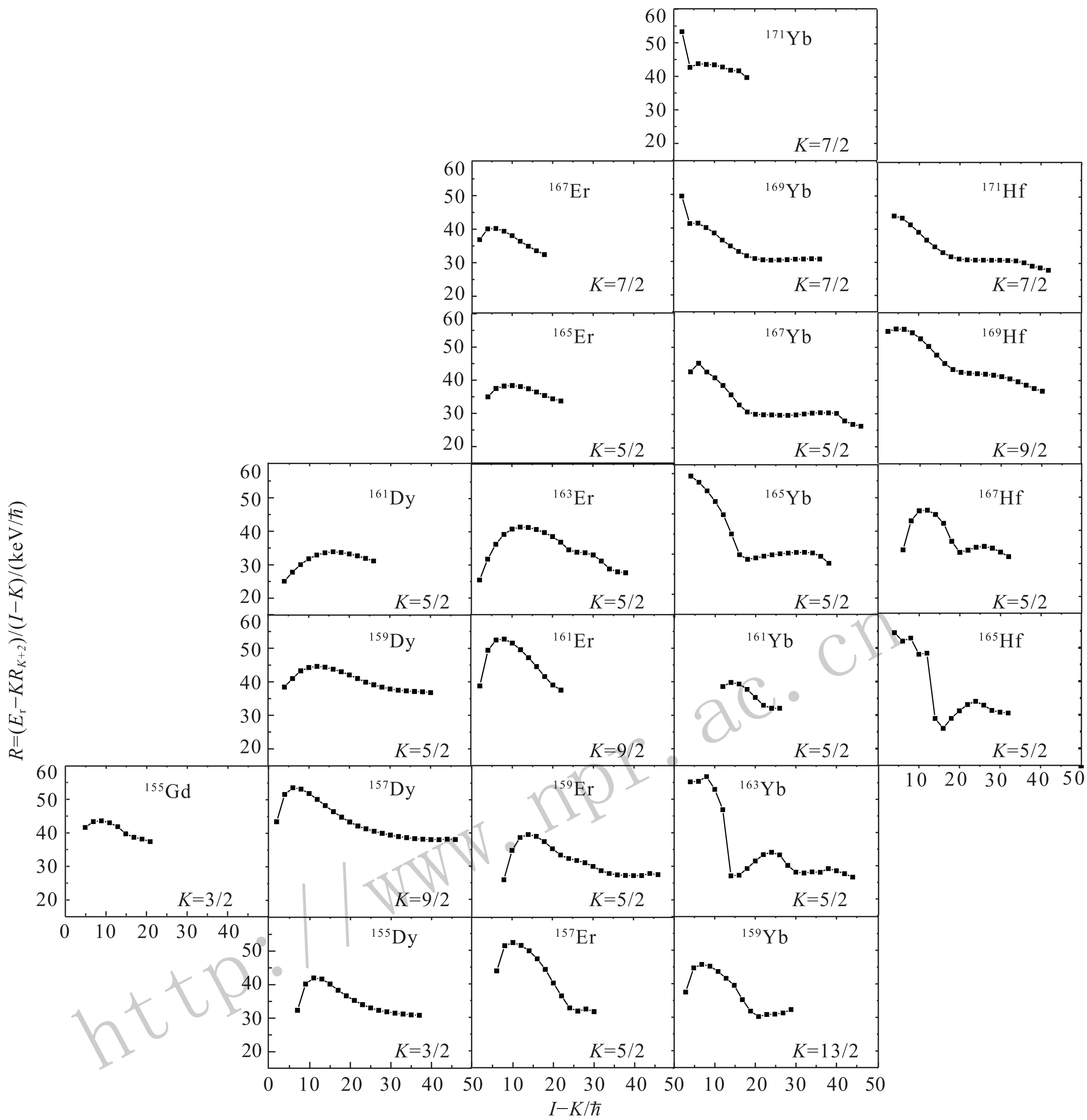

Fig. 2　The E-GOS curves of the odd-A nuclei in the mass region around $A=160$. The data are taken from Refs. [9–27]. Kdenotes the value of bandhead.

the vibrational region in these three nuclei. In other lighter Ytterbium and Hafnium isotopes shown in Fig. 2, however, the E-GOS plots of $^{159-163}$Yb and $^{165-169}$Hf also present the transition from rotation to vibration in some spin region.

3　Calculations and discussions

It is well known that the nuclear shapes are susceptible to the increased angular momentum. To gain an insight into the phase transitions and understand systematically the microscopic origin of these interesting phenomena, the total-routhian-surface (TRS) calculations was performed based on the nonaxial deformed Woods-Saxon potential[28] in a three-dimensional deformation space $(\beta_2, \gamma, \beta_4)$[29] . In this method, both monopole and quadrupole pairings are included[30–31]. To compare with the data, the relation between the frequency $\hbar\omega$ and angular momentum I can be expressed as[32]

$$\hbar\omega(I)=\frac{E(I+1)-E(I-1)}{I_x(I+1)-I_x(I-1)}, \tag{3}$$

where E and I_x are the level energy of state I and component of angular momentum along the axis of rotation, respectively. The I_x is given by[32]

$$I_x(I)=\sqrt{I(I+1)-K^2}\approx\sqrt{\left(I+\frac{1}{2}\right)^2-K^2}, \tag{4}$$

in which K is the bandhead spin. At a given frequency, the deformation of a state is determined by minimizing the calculated TRS.

The shape calculation results for two types of transition in the yrast bands of ^{157} Dy and ^{165}Yb were presented in Figs. 3 and 4, respectively. The calculations show that the low-lying configurations in ^{157}Dy exhibit a stable prolate deformation of $\beta_2 \approx$0.30 and a triaxiality parameter of $\gamma \approx 22.0^0$, and the rotation excitations are expected to be favored at low frequencies, which is consistent with the E-GOS curve characteristic of ^{157}Dy given in Fig. 2. At higher rotational frequency ($\hbar\omega >$0.5 MeV), an obvious reduction of the quadrupole deformation is observed by the theoretical prediction. The same result was reported in Ref. [33] according to the measurement of B(E2) values. Although the reduction of the deformation with increasing spin was predicted by the calculations, the minimum is also become more rigid given in Fig. 3. It indicates that the collective vibration is hard to develop, which is different from the E-GOS curve property observed in ^{157}Dy at higher spins. As shown in Fig. 4, the nucleus ^{165}Yb is predicted to be prolate with a quadrupole deformation of β_2= 0.266 and a triaxiality parameter of $\gamma = -120°$ at rotational frequency ω=0, which corresponds to ground state in ^{165}Yb. In addition, it should be pointed out that when the rotational frequency $\omega = 0$, *i.e.*, the nucleus is static, the case with $\gamma = -120°$ is equivalent to the one with $\gamma = 0°$, namely, axially symmetric prolate shape. As spin increasing, a stable prolate deformation was observed in ^{165}Yb with the rotational frequency of $\hbar\omega < 0.60$ MeV, whereas a distinct decrease in the quadrupole deformation and a triaxiality parameter of $\gamma = -7.525°$ were predicted at higher rotational frequency ($\hbar\omega$=0.80 MeV) by the calculations. Therefore, it is reasonable to expect that the yrast band of ^{165}Yb is in good rotational at low and medium angular momenta. In this work, however, when we discuss it in another way, *i.e.*, the E-GOS curve is used for the yrast band, an obvious evolution from vibrational to rotational structure is found in ^{165}Yb with increasing spin. This indicates that there is risk of simply assuming the rotational-based concepts over the entire spin range for nuclei which are not statically deformed.

As discussed above, it is found that the nuclear shape is strongly angular momentum dependent. Moreover, the energies of various levels in the rare-earth nuclei are considered to follow very closely the

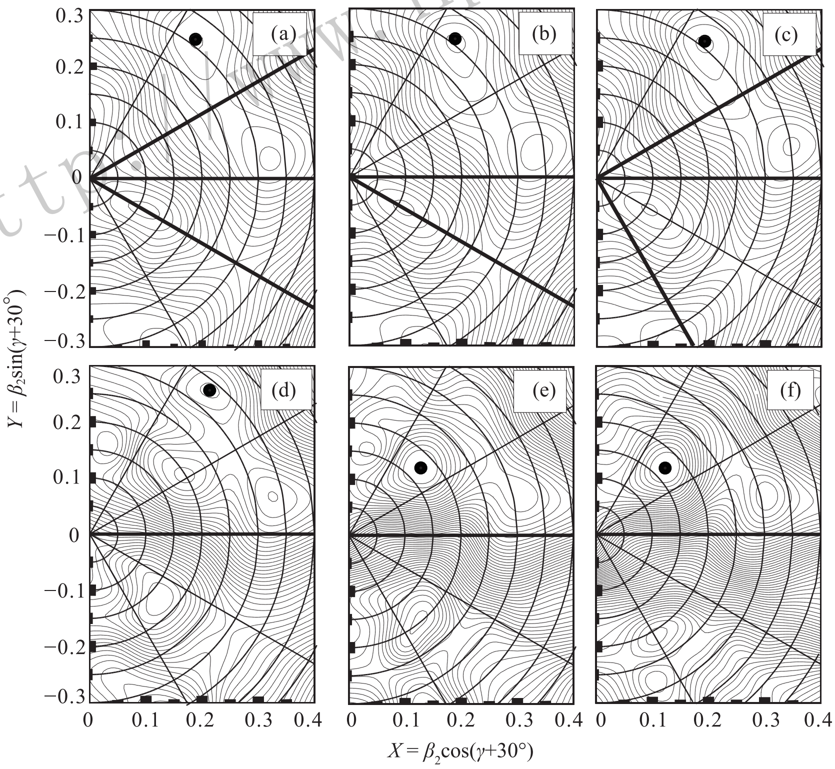

Fig. 3 Total routhian surfaces plots for the lowest $(\pi, \alpha) = (+, +1/2)$ configuration of ^{157}Dy. The energy contours are at 200 keV intervals. The deformation parameters for the individual minima are: (a) $\hbar\omega = 0.0$ MeV, $\beta_2 = 0.311$, $\beta_4 = -0.090$, and $\gamma = 22.838°$; (b) $\hbar\omega = 0.10$ MeV, $\beta_2 = 0.309$, $\beta_4 = -0.091$, and $\gamma = 22.499°$; (c) $\hbar\omega = 0.20$ MeV, $\beta_2 = 0.310$, $\beta_4 = -0.090$, and $\gamma = 21.756°$; (d) $\hbar\omega = 0.40$ MeV, $\beta_2 = 0.340$, $\beta_4 = -0.089$, and $\gamma = 22.604°$; (e) $\hbar\omega = 0.60$ MeV, $\beta_2 = 0.176$, $\beta_4 = -0.097$, and $\gamma = 13.175°$; (f) $\hbar\omega = 0.80$ MeV, $\beta_2 = 0.171$, $\beta_4 = -0.097$, and $\gamma = 13.765°$.

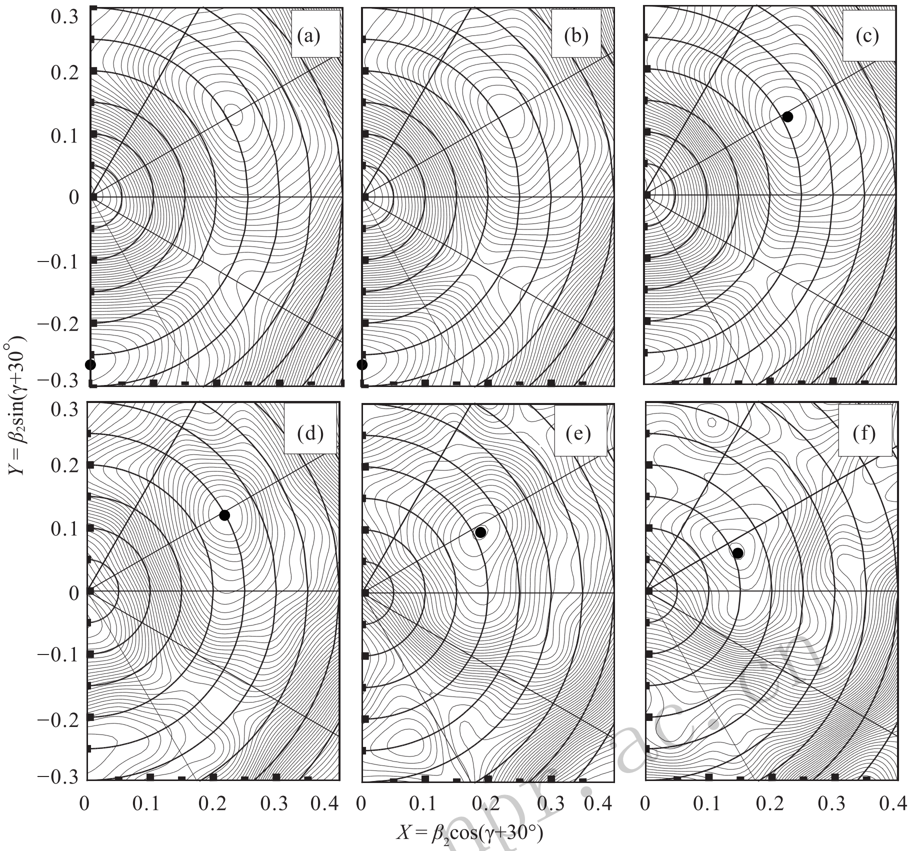

Fig. 4　Total Routhian surfaces plots for the lowest $(\pi,\ \alpha)$ = (-, +1/2) configuration of ^{165}Yb. The energy contours are at 200 keV intervals. The deformation parameters for the individual minima are: (a) $\hbar\omega=0.0$ MeV, $\beta_2=0.266$, $\beta_4=0.011$, and $\gamma=-120°$; (b) $\hbar\omega=0.10$ MeV, $\beta_2=0.266$, $\beta_4=0.012$, and $\gamma=-119.914°$; (c) $\hbar\omega=0.20$ MeV, $\beta_2=0.260$, $\beta_4=0.007$, and $\gamma=-1.46°$; (d) $\hbar\omega=0.40$ MeV, $\beta_2=0.249$, $\beta_4=-0.012$, and $\gamma=-1.166°$; (e) $\hbar\omega=0.60$ MeV, $\beta_2=0.211$, $\beta_4=-0.022$, and $\gamma=-2.896°$; (f) $\hbar\omega=0.80$ MeV, $\beta_2=0.158$, $\beta_4=-0.032$, and $\gamma=-7.525°$.

simple formula[7]

$$E_I=\frac{\hbar^2}{2J}I(I+1)-BI^2(I+1)^2, \tag{5}$$

where J is an effective moment of inertia. Generally, the rotational collective states are suggested to be described by the simpler formula[7]

$$E_I=\frac{\hbar^2}{2J}I(I+1). \tag{6}$$

The Eq. (6) can describe the rotational collective states well enough, especially for the lower values of I, based on the fact that the rotational spectra of a diatomic molecule with energies given by the formula are known to be exist[7]. But from this work, by using the E-GOS prescription, a clear structure evolution with increasing spin is found in the odd-A nuclei. So it's unreasonable to omit the second term of Eq. (5) for energy calculations in these nuclei if all spin states are considered.

The phase transition in rare-earth nuclei can be understood microscopically by the changes in the single-particle structure caused by the Coriolis force, which acts on the quasi-particles or nucleon pairs and leads to the alignment of the single-particle angular momenta along the rotation axis. As the influence of the Coriolis force on the orbits with high-j and small Ω value is the strongest, the alignment of $i_{13/2}$ neutrons and $h_{11/2}$ protons are expected to be favored in this mass region[33]. After band crossing, the wave functions of the excited states predominantly consist of maximally aligned quasiparticle orbitals. These quasiparticles may polarize the core to a rigid quadrupole deformation, and thus collective rotational motion would develop. In other words, the polarization effect from the aligned quasiparticles may be the reason of the transition from vibration to rotation discussed above. For the deformed nuclei, the vibrational excitations are considered to be far from the yrast states. In fast rotating nuclei, however, the rearrangement of the nuclear mass distribution caused by the alignment of the nucleon orbits and the centrifugal force acting would change the nuclear deformation and result in a loss of the axial symmetry of the nucleus. Thus, the collective vibration becomes favored over the rotation excitations, which may be applied to interpret the observation of the evolution from rotation to vibration in the odd-A rare-earth nuclei as a function of spin.

4 Conclusions

A simple prescription was used to distinguish vibrational from rotational regimes in the odd-A rare-earth nuclei. The characteristics of E-GOS curves of ^{155}Gd,$^{155-161}$Dy and $^{157-167}$Er suggest that these nuclei undergo a clear evolution from rotational to vibrational excitations along the yrast line with increasing angular momentum. For the ^{165,167}Yb and ^{171}Hf nuclei, however, the E-GOS curves of these yrast bands have the vibrational characteristic in the lower-spin region, whereas with higher spins it has a rotational pattern, and with spin above 35 there is again a change from the rotational to the vibrational region. The total-Routhian-surface calculations with nonaxial deformed Woods-Saxon potential were performed for the analysis of shape evolution occurring in ^{157}Dy and^{165}Yb. Comparison with the experimental data provides a consistent picture of the shape evolution in these nuclei in term of angular momentum. The current work also highlights the potential dangers of simply assuming the rotational-based concepts over the entire spin range. In this letter, we aim to discern the structure evolution in some mass region. The theoretical studies to describe such evolution from axial rotation to vibration with increasing spin in individual nucleus are beyond the scope of this work.

References:

[1] PIPIDIS A, RILEY M A, SIMPSON J, *et al.* Phys Rev C, 2005, **72**: 064307.

[2] FAISAL J Q, HUA H, LI X Q, *et al.* Phys Rev C, 2010, **82**: 014321.

[3] CAMPBELL D B, LAIRD R W, RILEY M A, *et al.* Phys Rev C, 2007, **75**: 064314.

[4] SUN Y, WALKER PHILIP M, XU F R, *et al.* Phys Lett B, 2008, **659**: 165.

[5] REGAN P H, BEAUSANG C W, ZAMFIR N V, *et al.* Phys Rev Lett, 2003, **90**: 152502.

[6] YANG J, WANG H L, CHAI Q Z, *et al.* Prog Theor Exp Phys, 2016, **063**: D03.

[7] SHEN S F, CHEN Y B, XU F R, *et al.* Phys Rev C, 2007, **75**: 047304.

[8] REGAN P H, WHELDON C, YAMAMOTO A D, *et al.* Acta Physica Polonica B, 2005, **36**: 1313.

[9] HAYAKAWA T, TOH Y, OSHIMA M, *et al.* Eur Phys J A, 2002, **15**: 299.

[10] CHEN L, ZHOU X H, ZHANG Y H, *et al.* Phys Re C, 2011, **83**: 034318.

[11] HAYAKAWA T, OSHIMA M, HATSUKAWA Y, *et al.* Nucl Phys A, 1999, **657**: 3.

[12] GRANJA C, POSPÍŠIL S, APRAHAMIAN A, *et al.* Phys Rev C, 2004, **70**: 034316.

[13] SUGAWARA M, MITARAI S, KUSAKARI H, *et al.* Nucl Phys A, 2002, **699**: 450

[14] ANDREA JUNGCLAUS, BINDER B, DIETRICH A, *et al.* Phys Rev C, 2003, **67**: 034302.

[15] SIMPSON J, BUTLER P A, FORSYTH P D, *et al.* J Phys G: Nucl Phys, 1984 **10**: 383.

[16] WANG S T, ZHOU X H, ZHANG Y H, *et al.* Phys Rev C, 2011, **84**: 017303.

[17] VLASTOU R, PAPADOPOULOS C T, SERRIS M, *et al.* Nucl Phys A, 1994, **580**: 133.

[18] BROCKSTEDT A, LYTTKENS-LINDÉN J, BERGSTRÖM M, *et al.* Nucl Phys A, 1994, **571**: 337.

[19] CROMAZ M, DEGRAAF J, DRAKE T E, *et al.* Phys Rev C, 1999, **59**: 2406.

[20] SCHMIDT K A, BERGSTRÖM M, HAGEMANN G B, *et al.* Eur Phys J A, 2001, **12**: 15.

[21] FEWELL M P, JOHNSON N R, MCGOWAN F K, *et al.* Phys Rev C, 1988, **37**: 101.

[22] SELIN E, HJORTH S A, ANDRYDE H. Physica Scripta, 1970, **2**: 181.

[23] ARCHER D E, RILEY M A, BROWN T B, *et al.* Phys Rev C,1998, **57**: 2924.

[24] BLUME K P, HUBEL H, MURZEL M, *et al.* Nucl Phys A, 1987, **464**: 445.

[25] CULLEN D M, REED A T, APPELBE D E, *et al.* Nucl Phys A, 2000, **673**: 3.

[26] ROY N, JÓNSSON S, RYDE H, *et al.* Nucl Phys A, 1982, **382**: 125.

[27] RICHTER L, BACKE H, KANKELEIT E, *et al.* Phys Lett B, 1977, **71**: 74.

[28] NAZAREWICZ W, DUDEK J, BENGTSSON R, *et al.* Nucl Phys A, 1985, **435**: 397.

[29] NAZAREWICZ W, WYSS R, JOHNSON A. Phys Lett B, 1989, **225**: 208.

[30] SATULA W, WYSS R. Phys Rev C, 1994, **50**: 2888.

[31] XU F R, SATULA W, WYSS R. Nucl Phys A, 2000, **669**: 119.

[32] DE VOIGT M J A, DUDEK J, SZYMAŃSKI Z. Rev Mod Phys, 1983, 55:949.

[33] EMLING H, GROSSE E, KULESSA R, *et al.* Nucl Phys A, 1984, **419**: 187.

稀土区奇 A 核晕带存在从转动到振动形状演化的直接证据

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摘要： 原子核的形状演化效应是核结构研究的重要基础问题之一。通常认为，$A=160$ 质量区的奇 A 核位于大形变核区域，它们的激发态能谱将呈现出典型的转动激发特征。然而，基于 E-GOS 曲线方法，发现随着角动量的增加，该质量区奇 A 核的晕带具有显著地从转动激发演化成为振动激发的形状演化现象。此外，为深入理解原子核形状演化的微观机制，采用 Total-Routhian-Surface (TRS) 方法针对稀土区的奇 A 核进行了理论计算，结果表明，^{165}Yb 和 ^{157}Dy 同位素在低激发态时具有稳定的长椭形变，当角动量大于 0.50 MeV 后，核芯的四极形变显著减小并开始产生三轴形变。

关键词： 相变；E-GOS 曲线；总罗斯面

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