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Study of the Systematic Errors Introduced by the Statistical Model in the Measurement of ^{12}C+^{13}C Fusion Reaction

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Abstract: The online γ-rays and activity decay of ^{24}Na residual produced in the ^{12}C+^{13}C reaction are both measured using thick and thin targets in the center-of-mass energy of 4∼6 MeV. The total fusion cross section is derived from the γ-ray yields using the theoretical correction calculated by TALYS. Comparing the cross sections obtained by different methods in previous experiments, the systematic errors in the total cross section determination by branching the online γ-ray yields and the ^{24}Na channel cross section are both determined to be 14%.

Key words: fusion reaction; statistical model; characteristic γ measurement; offline activity measurement

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1 Introduction

The ^{12}C+^{12}C fusion reaction is one of the most important reactions in nuclear astrophysics[1–2]. The temperature of carbon burning varies from 0.8∼1.2 GK, corresponding to 1∼3 MeV[2]. Due to the extremely low cross section in the astrophysical energy region, all measurements have been limited to the energy above 2.1 MeV during the past several decades[3]. The cross section at lower energies has to be based on the theoretical extrapolation[4–7]. However, due to the strong narrow resonances in ^{12}C+^{12}C reaction, theoretical predictions become very difficult and are not consistent with each other[8]. In contrast to the complicated resonances in ^{12}C+^{12}C, the similar isotope system ^{12}C+^{13}C behaves more smoothly[9–11]. Therefore, the measurement of ^{12}C+^{13}C can be used to constrain the cross section of ^{12}C+^{12}C and test the extrapolation models[12–13].

Until now, three different methods, online characteristic γ-rays[14], on-line total γ-ray yield[15] and activity detection of the residual nucleus ^{24}Na[13], have been used to determine the fusion cross sections of ^{12}C+^{13}C at sub-barrier energies with the aid of statistical model. The fusion cross sections have been measured down to $E_{\rm c.m.}$=3.3 MeV through the online measurements by Dayras *et al.*[14] and Dasmahapatra *et al.*[15]. At the lower energies, due to very low yields and ambient background, only activity measurement has been done by Notani *et al.*[12] and our group[13]. By exploiting an underground counting lab with an ultra-low background, we pushed the ^{12}C+^{13}C fusion cross section down to 0.9 nb with a statistical error less than 30%. In this measurement, the total fusion cross section was just derived from the ^{12}C(^{13}C,p)^{24}Na cross section depending on the theoretical correction for ^{24}Na channel. Therefore, detailed studies on the systematic uncertainty from the statistical model calculations are needed in ^{12}C+^{13}C. This is more important for the activity experiment at deep sub-barrier energies, where branching ratio is essential for the determination of total fusion cross section.

In this work, online γ-rays and activity decay of ^{24}Na residual are both measured in the energy range of 4∼6 MeV. In the meantime, TALYS calculations

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have been performed and discussed. Together with previous measurements, the systematic uncertainty is investigated.

2 $^{12}C+^{13}C$ experiment

The ^{13}C beam was provided by the 3 MV Tandem accelerator of Sichuan University[16]. The energy ranges covered were $E_{\rm lab}$=8.6∼12 MeV in steps of 200 keV and beam currents were typically from 1.5∼5.0 eμA. Two types of targets, thin self-supporting carbon targets with a thickness of 20 μg/cm^2 and 1 mm thick targets, were used in the measurement. The fusion cross sections of $^{12}C+^{13}C$ were obtained by the online characteristic γ-ray technique. One of the products ^{24}Na, via the 1-proton evaporation channel of the compound nucleus, is unstable and beta decays with a half-life of 14.997 h. The offline activity measurement of ^{24}Na was performed for some thin targets and most of the thick targets, where a copper sheet was placed 1 cm away from the thin target to collect the products.

Online γ-rays were detected by a single HPGe detector located at zero degree to the beam direction. The distance between target and Ge crystal is 5 cm where the summing effect from cascade γ-rays was negligible. After the irradiation, the target samples would be taken to the counting area for the activity measurement of the residual ^{24}Na. In both measurements, the target sample and the Ge crystal were well shielded with lead bricks. The prominent γ-ray transitions in the determination of the cross sections of different reaction channels are listed in Table 1 and one typical γ-ray spectrum obtained at $E_{\rm c.m.}$=5.76 MeV is presented in Fig. 1. The yields were deduced following the standard procedure for the activity measurement,

Table 1 The prominent γ-ray transitions detected in the experiment.

Number	E_γ/MeV	Transition	Reaction
1	0.351	^{21}Ne(5/2$^+$→3/2$^+$)	^{12}C(^{13}C, α)
2	0.440	^{23}Na(5/2$^+$→3/2$^+$)	^{12}C(^{13}C, pn)
3	0.472	^{24}Na(1$^+$→4$^+$)	^{12}C(^{13}C, p)
4	1.369	^{24}Mg(1$^+$→0$^+$)	^{12}C(^{13}C, n)

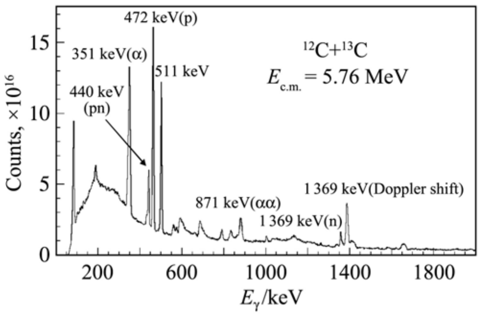

Fig. 1 A γ-ray spectrum obtained in online measurement of $^{12}C+^{13}C$ with thin target.

in which the beam current fluctuation, cooling time were also considered[17–18] and the differential method was used to deduce the cross sections from the thick-target yields (see Ref. [3] and references therein). Our measurement includes a 9% systematic uncertainty which primarily results from the uncertainties in the beam current (5%), detector efficiency (3%) and thickness of target (5%).

The fusion cross sections of $^{12}C+^{13}C$ reaction are deduced by correcting the γ-ray yields with the help of theoretical branching ratios calculated by Hauser-Feshbach model[19]. These corrections required a knowledge of the population of the compound states and their decay properties to obtain the branching ratios for all the observed channels. In this work, TALYS code[20] was used to calculate the branching ratios in the range of 2 to 6 MeV for $^{12}C+^{13}C$ reaction. The optical model parameters used in TALYS are listed in Table 2.

Table 2 Optical model parameters used in this work*.

Channel	V_1/MeV	$R_{\rm R}$/fm	$a_{\rm R}$/fm	W_1/MeV	$R_{\rm I}$/fm	$a_{\rm I}$/fm
^{24}Mg+n	50.02	3.375	0.676	6.800	3.740	0.540
^{24}Na+p	57.64	3.366	0.674	10.735	3.740	0.536
^{21}Ne+α	55.34	3.654	0.675	22.990	4.127	0.535

* The Woods-Saxon optical model is chosen in our TALYS calculationin[20], and the parameters for the neutron and proton channels are specifically tuned to reproduce the partial cross sections in Ref. [14]. The nuclear level density deduced from the microscopic Hartree-Fock approach, which has been adopted in the RIPL[21] database, is taken into account. The width fluctuation correction factors obtained from the Moldauer model are considered. In TALYS calculation, the initial population of the excited compound nucleus ^{25}Mg formed via $^{12}C+^{13}C$ fusion is provided by the external file, and the light particles including neutron, proton, deuteron, triton, ^{3}He, ^{4}He and photon[22] are taken into account in the decay of ^{25}Mg.

3 Results and discussion

Fig. 2(a) shows the independent determinations of the modified S factors (S^*) for each channel and also the comparison with ESW model[15]. The S^* is defined as $S^*(E)=\sigma(E)E{\rm e}^{(87.21/\sqrt{E}+0.46E)}$[12]. The Equivalent Square Well (ESW) is a simple model consisting of 3 parameters, radius, real and imaginary potential. The prediction of this model was made based on the Dayras *et al.* measurement. The experimental cross section of $^{12}C+^{13}C$ was deduced independently from the 1∼4 transitions listed in Table 1 and ^{24}Na decay by dividing their branching ratios. The good agreement of the total S^* factors inferred from the four online γ-ray transitions shows the accuracy of the branching ratio calculations and the present method. In the meantime, the relative agreement between the total S^* factors inferred from the 472 keV transition

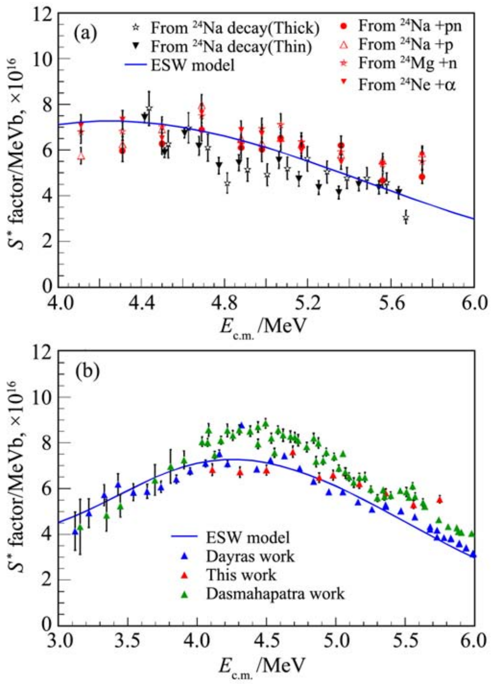

Fig. 2 (color online) Comparison of the S^* factor of ^{12}C+ ^{13}C obtained from different channels and different measurements. (a) The S^* factor of ^{12}C+^{13}C derived from the major reaction channels. The cross section is defined as $\sigma=\sigma_\gamma/(R^x_\gamma\cdot R^x)$, where σ_γ is the measured γ-ray yield of the x channel, R^x_γ represents the γ-ray population probability in the x channel and R^x is the branching ratio of the compound nucleus decaying to the x channel. (b) Comparison of S^* factors in different measurements. The total fusion cross section of this work is defined as $\sigma_{\rm tot}=(\sigma_{\rm n}+\sigma_{\rm p}+\sigma_{\rm np}+\sigma_\alpha)/(R^{\rm n}+R^{\rm p}+R^{\rm np}+R^\alpha)$, where $\sigma_{\rm x}$ represents the cross section of the x channel.

in ^{24}Na and the decay of ^{24}Na with both thick and thin targets, also provides an important check on the TALYS correction used in ^{24}Na channel. The fluctuation of data points from ^{24}Na decay is most likely from the differentiation of the thick target yields.

In principle, one needs to determinate the total fusion cross section by summing the partial cross sections of all the exit channels. However, if the production cross section for an individual γ-ray is very small, it will be very difficult to obtain the partial cross section of this channel with a good accuracy. A reasonable way is to first obtain the partial cross sections of the major reaction channels with the observed strong transitions and then determine the total fusion cross section with the sum of the cross sections of the major channels and the ratio of the weak unmeasured channels. In this work, the total fusion cross section of ^{12}C+^{13}C reaction is obtained by branching and summing the p, n, α, pn exit channels, which are populated intensely and free of background. The S^* factor is shown in Fig. 2(b), and compared with the results of Dayras *et al.* and Dasmahapatra *et al.* These three data sets agree with each other within 17%.

In Fig. 2(b), Dayras *et al.*[14] obtained the total fusion cross section using the yields of six characteristic γ-rays and the statistical model correction, very similar to the approach described above. In the work of Dasmahapatra *et al.*, all the γ transitions were measured and summed via the NaI summing detectors. Total fusion cross section was deduced after correction for the population probability of different bound states. Although the statistical model calculations were used in these three data sets, the branching corrections are very different. Therefore, the comparison among these data enables us to estimate the systematic uncertainty of the corrections used in the online γ-ray measurements.

Before the quantified comparison was made, the experimental data and branching ratio calculations by Dayras *et al.* were evaluated again in this work. Fig. 3(a) shows the S^* factors of ^{12}C+^{13}C determined independently by p, n, α channels in Ref. [14]. The S^* factor inferred from the γ transition of ^{24}Mg is 30% lower than that from ^{24}Na and ^{21}Ne in the region from 4 to 6 MeV. In order to resolve this discrepancy, we re-calculated the S^* factors of ^{12}C+^{13}C using the individual cross sections of the p, n, α channels corrected by our TALYS calculation. The new results are pre-

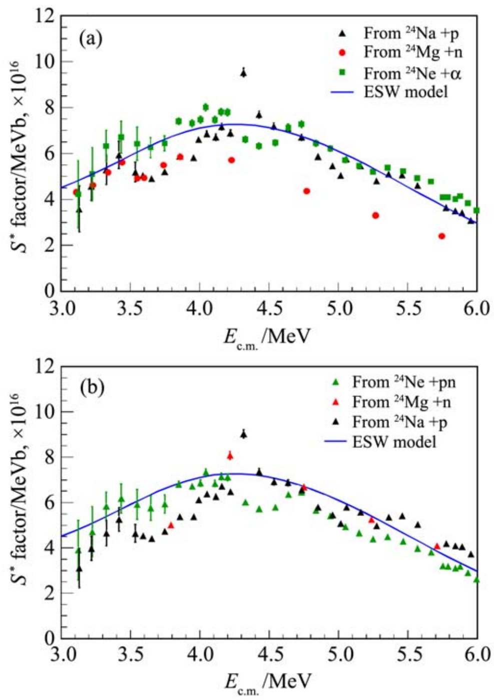

Fig. 3 (color online) Comparison of the S^* factors of ^{12}C+^{13}C using different branching ratio calculations based on the measurement by Dayras *et al.*[14]. The S^* factors are inferred independently from the γ-ray yields in ^{24}Na, ^{24}Mg and ^{21}Ne using the branching ratios in Ref. [14] (a) and our TALYS calculations (b).

sented in Fig. 3(b). The good agreement among the three channels shows the self-consistency of the correction procedure and our TALYS calculation. Consequently, the total cross section was updated and used in the following discussion.

The comparisons of the S^* factors obtained from Dayras, Dasmahapatra and this work are shown in Fig. 4, in which the data by Dasmahapatra *et al.* are used as the reference. The ratios for our data and that of Dayras *et al.* both exhibit obvious fluctuations, reflecting the absolute experimental error. The ratio from constant fitting of both data is 0.89±0.09, The fluctuation (0.09) and the deviation of the ratio value from 1 result from the model uncertainty and experimental uncertainty. As a conservative estimation, we combine the central deviation (0.11) with the standard deviation (0.09) and recommend 14% as the systematic uncertainty for the branching ratio calculated by statistical model. This uncertainty is also consistent with the value proposed in our previous study at relatively higher energies[23].

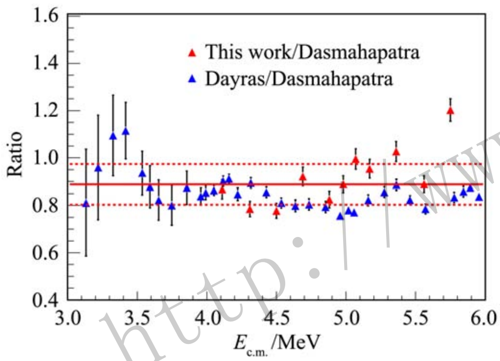

Fig. 4 (color online) Comparison of S^* factors derived from the $^{12}C+^{13}C$ total fusion cross sections. The red solid line and dash lines represent the constant fit (0.89) and 1σ deviation (±0.09), respectively.

In the lower energy experiments ($E_{c.m.}$ <3.2 MeV) by us[13] and Notani *et al.*[12], only activity measurements were used to obtain the $^{12}C(^{13}C,p)^{24}Na$ channel cross sections and the total cross sections are derived only from this channel based on the statistical model calculations. Therefore, a reliable branching ratio of the proton channel with a quantified uncertainty is highly needed. In Fig. 5, the experimental branching ratios are obtained by dividing the cross section of the ^{24}Na channel in different experiments to the total cross sections obtained by Dasmahapatra *et al.*[15]. The data fluctuation around the TALYS calculation is 14%, reflecting the systematic uncertainties from the measurements and statistical model. We adopt 14% as the systematic errors for the ^{24}Na branching ratio obtained from statistical model.

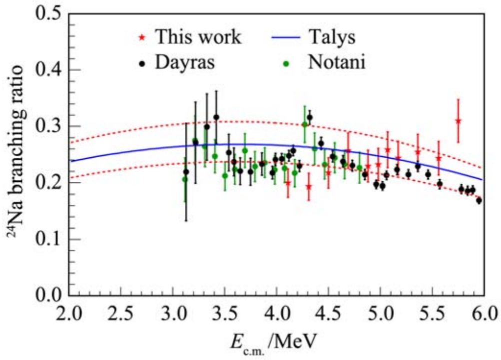

Fig. 5 (color online) Branching ratios of ^{24}Na channel. The branching ratios are defined as $R=\sigma_{^{24}\mathrm{Na}}/\sigma_{\mathrm{total}}$, where the result of Dasmahapatra *et al.*[15] is adopted as the total cross section.

As a conclusion, online characteristic γ-rays and the decay of the residual ^{24}Na in $^{12}C+^{13}C$ reaction were both measured using thin and thick targets. The total cross section of $^{12}C+^{13}C$ is obtained and compared with the results of Dayras *et al.* and Dasmahapatra *et al.* The systematic error of the branching ratios converting the online γ-ray yields to the total cross section is analyzed and determined to be 14%. In the meantime, the uncertainty of the total cross section singly inferred from the decay measurement of ^{24}Na channel is discussed and a value of 14% is recommended.

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$^{12}C+^{13}C$熔合截面测量中统计模型引入的系统误差研究

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摘要： 在本工作中，我们分别采用薄靶和厚靶实验技术，测量了$^{12}C+^{13}C$反应在质心系能量4~6 MeV的熔合截面。实验得到了$^{12}C+^{13}C$反应的在线γ产额和离线的^{24}Na活度，利用TALYS统计模型给出的反应道分支比，导出了熔合反应的总截面。通过对比不同实验得到的总截面，定量研究了统计模型修正所带来的系统误差：在线γ分支比修正引入的系统误差为14%；由离线^{24}Na活度测量得到总截面时，^{24}Na分支比修正带来的系统误差也为14%。

关键词： 熔合反应；统计模型；特征γ测量；离线活度测量

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