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Contribution of Deuteron Breakup Effect in Outgoing Proton Energy Spectra*

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Abstract: Based on the continuum discretized coupled channels(CDCC) theory, and the hypothesis that the residual nucleus locates not only in bound state but also in a series of excited states, the contribution of deuteron breakup effect in outgoing proton energy spectra are studied in the present work. And the results can basically explain the experimental results that a peak structure is appeared in the outgoing proton energy spectra at about half of the incident energy. The results of present calculation generally agree with those obtained from C. Kalbach's empirical formulaum.

Key words: deuteron; breakup effect; CDCC theory; outgoing proton energy spectrum

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1 Introduction

Deuteron is a weakly bound nucleus which can easily break into proton and neutron in nuclear collisions, and its breakup effects in elastic scattering angular distributions and reaction cross sections are systematically studied for many target nuclei in a wide energy region[1, 2]. The contributions from the closed channels to the total reaction and breakup cross sections, and angular distributions of elastic scattering are also seriously discussed[3]. Some researches[4, 5] point out that for all the nuclei studied, the proton energy spectra show large deuteron-breakup peaks centered at about half of the incident deuteron energy, however the evaporation model and exciton model of nuclear reaction cannot give this peak structure. In order to obtain this peak structure, C. Kalbach[6, 7] introduced transfer and knock out reaction mechanism in deuteron induced reaction based on exciton model, but this treatment still cannot produce the peak structure. In order to reproduce the experimental data, C. Kalbach adopted an empirical formula of Gaussian shape in pre-equilibrium process, and used it to describe the deuteron breakup effect in outgoing proton or neutron energy spectra.

The purpose of this paper is to research microscopically the phenomenon that the proton energy spectra show large deuteron-breakup peaks centered at about half of the incident deuteron energy, and to provide a theoretical foundation for C. Kalbach's empirical formula. Based on contimuum discretized coupled channels (CDCC) theory, for deuteron stripping reaction, the present work supposes that the residual nucleus is situated not only in bound state but also in a series of excited states, and a code CDCCOM (Optical Model in CDCC theory) is written based on this hypothesis.

This paper is arranged as follows. In Section 2, we begin with the introduction of CDCC theory and the code CDCCOM briefly; we study the transition matrix element of stripping reaction expressed by three body wave function in Section 3;

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in Section 4，the transition matrix element is given；Section 5 is the result，and Section 6 is a summary.

2　CDCC Theory and Code CDCCOM

The CDCC approach[8] is based on three body theory，and for deuteron induced reaction，it considers the interactions among p，n and target nucleus. The total energy of this three body system keeps fixed in the whole reaction process. The code CDCCOM[2，3] is based on CDCC theory and written by present authors to study the deuteron breakup effects on reaction cross sections etc. and in CDCCOM，the model space is truncated as $l=2$ and the number of discretized coupled channels is ten，where l is relative orbital angular momentum quantum number of the p to n，namely，only considering the contributions of s-wave function and d-wave function is enough.

3　Transition Matrix Element of Stripping Reaction

For A(d，p)B reaction，the general form of transition matrix element is

$$T(\mathrm{d},\ \mathrm{p}) = \langle \psi_{\beta}^{(-)} | V_{\mathrm{pn}} + U_{\mathrm{pA}} - \overline{U}_{\mathrm{pB}} | \Phi_{\alpha}^{(+)} \rangle \ , \quad (1)$$

where $\Phi_{\alpha}^{(+)}$ is the total wave function of the three body system，$\psi_{\beta}^{(-)}$ is the wave function in outgoing channel，V_{pn} is the interaction potential between p and n，U_{pA} and $\overline{U}_{\mathrm{pB}}$ are the optical potential of proton-target and proton-residual nucleus，respectively. When the mass of target nucleus is far larger than the mass of proton or neutron，we can believe that U_{pA} is equal to $\overline{U}_{\mathrm{pB}}$ approximately，then the residual interaction potential is only V_{pn}. Supposing the deuteron incident direction is the forward direction of z axis，and J denotes the total angular momentum quantum number of the A+2 system，M is the total angular momentum projection quantum number，L is the orbital angular momentum quantum number of the p-n pair centre of mass moving against target nucleus，we know that if $M\neq 0$，$Y_{JM}(0,\ \phi)=0$；and only if $M=0$，$Y_{JM}(0,\ \phi)=\sqrt{(2J+1)/4\pi}$. Then the J component of the transition matrix element of stripping reaction A(d，p)B is

$$T^{J}(\mathrm{d},\ \mathrm{p}) = \int [\psi_{lj}(\boldsymbol{r}_{\mathrm{pB}}) \otimes \psi_{l'j'}(\boldsymbol{r}_{\mathrm{nA}})]_{J0} \times V_{\mathrm{pn}}(r)\psi_{J_{\max}}^{(+)}(\boldsymbol{R},\ \boldsymbol{r})\mathrm{d}\boldsymbol{R}\mathrm{d}\boldsymbol{r} \ , \quad (2)$$

where $\boldsymbol{R}=(\boldsymbol{r}_{\mathrm{pA}}+\boldsymbol{r}_{\mathrm{nA}})/2$，$\boldsymbol{r}=\boldsymbol{r}_{\mathrm{pA}}-\boldsymbol{r}_{\mathrm{nA}}$，and the outgoing wave function is：

$$[\psi_{\mathrm{lj}}(\boldsymbol{r}_{\mathrm{pB}}) \otimes \psi_{l'j'}(\boldsymbol{r}_{\mathrm{nA}})_{J0} = \sum_{j=|J-j'|}^{J+j'} \sum_{m_j=-j}^{j} C_{j,\ m_j,\ j',\ -m_j}^{J0} \times \sum_{l=j\pm 1/2} \sum_{\mu=\pm 1/2} a_{l,\ m_j-\mu}^{*} \times C_{l,\ m_j-\mu,\ 1/2,\ \mu}^{jm_j} C_{l',\ -m_j+\mu,\ 1/2,\ -\mu}^{j'-m_j} \times u_{\mathrm{lj}}(r_{\mathrm{pB}}) u_{l'j'}(r_{\mathrm{nA}}) \times Y_{l,\ m_j-\mu}^{*}(\Omega_{\mathrm{pB}}) Y_{l',\ -m_j+\mu}^{*}(\Omega_{\mathrm{nA}}) \ , \quad (3)$$

where $\psi_{l'j'}(\boldsymbol{r}_{\mathrm{nA}})$ is the bound state of the residual nucleus B with definite quantum numbers $l'j'$，and u_{lj} and $u_{l'j'}$ are the radial wave functions.

The total wave function of the three body system $\Phi_{\alpha}^{(+)}$ can be written as

$$\psi_{J_{\max}}^{(+)}(\boldsymbol{R},\ \boldsymbol{r}) = \frac{a_{J0}}{R}[\phi_0(r)Y_{00}(\Omega_r) \times Y_{J0}(\Omega_R)\chi_J(P_0,\ R) + \sum_{l''=0,\ 2} \sum_{L=|J-l''|}^{J+l''} \sum_{i=1}^{NK=10} \phi_{l''}(k_i,\ r)\chi_{l''LJ}(P_{k_i},\ R) \times \sum_{m''=-l''}^{l''} C_{l'',\ m'',\ L,\ -m''}^{J0} Y_{l''m''}(\Omega_r) Y_{L-m''}(\Omega_R)] \ , \quad (4)$$

where $\phi_0(r)$ is ground state wave function of deuteron，$\phi_l(k_i,\ r)$ are the continuum discretized state wave functions of p-n pair with linear momentum $\hbar k_i$ and orbital angular momentum $l\hbar$.

The χ are coupled channel wave functions，they are calculated with P3C5[9] algorithm in CDCCOM. The predominance of the code CDCCOM is that it includes not only the coupling of the continuum states of p-n pair themselves，but also that of the ground state of deuteron. $\chi_J(P_0,\ R)$ expresses the p-n c. m. motion with momentum $\hbar P_0$ in the elastic channel，and $\chi_{lLJ}(P_{k_i},\ R)$ expresses the p-n c. m. motion with momentum $\hbar P_{k_i}$ in continuum discretized breakup channels. The index i denotes the discretized state number of p-n c. m. moving against target nucleus in breakup channels. The k_i and P_{k_i} satisfy the total energy conservation $E=(\hbar^2 P_{k_i}^2/2\mu_R)+(\hbar^2 k_i^2/2\mu_r)$, where μ_R and μ_r is the d-A and p-n reduced mass, respectively. The coefficients in formula (3) and (4) are

$$a^*_{lm}=\sqrt{4\pi}\,k_{\mathrm{pB}}^{-1}\,i^{-l}\,Y_{lm}(\hat{k}_{\mathrm{pB}});$$

$$\begin{aligned} a_{J0}&=\sqrt{4\pi}\,\lambda_{\mathrm{d}}^{-1}\,i^{J}\,Y_{J0}\,(\hat{\lambda}_{\mathrm{d}})^* \\ &=\lambda_{\mathrm{d}}^{-1}\,i^{J}\sqrt{2J+1}\ . \end{aligned} \tag{5}$$

If the $V_{\mathrm{pn}}(r)$ is taken as zero range potential, namely $V_{\mathrm{pn}}(r)=V_{\mathrm{pn}}^0\delta(r)$, then

$$\int Y_{l''m''}(\Omega_r)\,\mathrm{d}\Omega_r=\sqrt{4\pi}\,\delta_{l''0}\,\delta_{m''0}\,, \tag{6}$$

and a symbol W_{pn}^0 is introduced as

$$-V_0\int_0^{r_{\max}} \mathrm{e}^{-(r/r_0)^2}\times \left[\phi_0(r)+\sum_{l=0,\,2}\sum_{i=1}^{NK=10}\phi_l(k_i,\ r)\right]r^2\,\mathrm{d}r\rightarrow W_{\mathrm{pn}}^0\quad . \tag{7}$$

According to angular momentum theory of M. E. Lose, we obtain:

$$\int Y_{L0}(\Omega_R)\,Y^*_{l,\,m_j-\mu}(\Omega_R)\,Y^*_{l',\,-m_j+\mu}(\Omega_R)\,\mathrm{d}\Omega_R= \sqrt{\frac{(2l+1)(2l'+1)}{4\pi(2L+1)}}\,C^{L0}_{l,\,0,\,l',\,0}\,C^{L0}_{l,\,m_j-\mu,\,l',\,-m_j+\mu}\quad . \tag{8}$$

The C-G coefficients in above formula are not equal to zero only if the requirements $\Delta(ll'L)$ and $l+l'+L=$ even numbers are satisfied. If the center of mass of target nucleus is taken as the coordinate origin, then $r_{\mathrm{A}}=0$ and $\boldsymbol{r}=\boldsymbol{r}_{\mathrm{p}}-\boldsymbol{r}_{\mathrm{n}}=0$ for zero range approximation, and then:

$$\begin{aligned} \boldsymbol{R}&=\frac{\boldsymbol{r}_{\mathrm{p}}+\boldsymbol{r}_{\mathrm{n}}}{2}=\boldsymbol{r}_{\mathrm{n}}\,; \\ \boldsymbol{r}_{\mathrm{nA}}&=\boldsymbol{r}_{\mathrm{n}}-\boldsymbol{r}_{\mathrm{A}}=\boldsymbol{r}_{\mathrm{n}}=\boldsymbol{R}\ ; \\ \boldsymbol{r}_{\mathrm{pB}}&=\boldsymbol{r}_{\mathrm{p}}-\boldsymbol{r}_{\mathrm{B}}=\boldsymbol{r}_{\mathrm{p}}-\frac{m_{\mathrm{n}}\boldsymbol{r}_{\mathrm{n}}+m_{\mathrm{A}}\boldsymbol{r}_{\mathrm{A}}}{m_{\mathrm{n}}+m_{\mathrm{A}}} \\ &=\frac{m_{\mathrm{A}}\boldsymbol{R}}{m_{\mathrm{n}}+m_{\mathrm{A}}}=Q\boldsymbol{R}\ . \end{aligned} \tag{9}$$

According to the above formula we know $\Omega_{\mathrm{pB}}\approx\Omega_{\mathrm{nA}}\approx\Omega_R$.

With formula (3), (4), (6), (7), (8), and finished the integrals with respect to Ω_r and Ω_R, the formula (2) can be written as:

$$\begin{aligned} T^J(\mathrm{d},\ \mathrm{p})={}&a_{J0}W_{\mathrm{pn}}^0\int_0^{R_{\max}}R\,\mathrm{d}R\times \\ &\sum_{i=0}^{Nk=10}\phi_0(k_i,\ r)\,\chi_J(P_{k_i},\ R)\,u_{l'j'}(R)\times \\ &\sum_{j=|J-j'|}^{J+j'}\ \sum_{m_j=-j}^{j}C^{J0}_{j,\,m_j,\,j',\,-m_j}\sum_{l=j\pm1/2}u_{lj}(QR)\times \\ &\sum_{\mu=\pm1/2}a^*_{l,\,m_j-\mu}\,C^{jm_j}_{l,\,m_j-\mu,\,1/2,\,\mu}\,C^{j,\,-m_j}_{l',\,-m_j+\mu,\,1/2,\,-\mu}\times \\ &\sqrt{\frac{(2l+1)(2l'+1)}{(2J+1)}}\,C^{J0}_{l,\,0,\,l',\,0}\,C^{J0}_{l,\,m_j-\mu,\,l',\,-m_j+\mu}\quad . \end{aligned} \tag{10}$$

Based on the above transition matrix elements $T^J(\mathrm{d},\ \mathrm{p})$, the differential cross section is:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\mu_{\mathrm{dA}}\mu_{\mathrm{pB}}}{(2\pi\hbar^2)^2}\,\frac{k_{\mathrm{pB}}}{\lambda_{\mathrm{d}}}\left|\sum_J T^J(\mathrm{d},\ \mathrm{p})\right|^2, \tag{11}$$

and cross section of stripping reaction is:

$$\mathrm{d}\sigma=\int\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\,\mathrm{d}\Omega\ . \tag{12}$$

4　Transition Matrix Element in Present Hypothesis

For the A(d, p)B stripping reaction, the outgoing wave function(formula (3)) includes $\psi_{lj}(\boldsymbol{r}_{\mathrm{pB}})$ which is the proton outgoing wave function moving against residual nucleus B, and $\psi_{l'j'}(\boldsymbol{r}_{\mathrm{nA}})$ which is the bound sates wave function of neutron-target nucleus A. The bound state $\psi_{l'j'}(\boldsymbol{r}_{\mathrm{nA}})$ has its own total angular momentum, orbital angular momentum and binding energy B_{nA}(usually several MeV). If this is true, according to energy conservation of p-B system, for a certain deuteron incident energy E, the outgoing proton energy spectra only at about $E_{\mathrm{p}}=E-B_{\mathrm{nA}}$ can be calculated. However, experimental results show that for a target nucleus the contribution of deuteron breakup effects in outgoing proton energy spectra exists in whole energy range.

In order to study the breakup effects in outgoing proton energy spectra in whole energy range, we suppose that besides the bound state of neutron and target, they can also form a series of excited state with continuous positive energy, then according to the energy conservation of p-B system, the contribution of breakup effect in outgoing proton energy spectra will be obtained in whole energy range.

If the excitation energy of residual nucleus (n-A system) is U, its wave function can be expanded with different partial wave function $u_{l'j'}(R)$. In normal temperature Fermi parameterized level density formula, the distribution function of angular momentum is

$$R(j', U) = 2j' + 1/2\sigma_s^2 \exp\left[-\frac{(j'+1/2)^2}{2\sigma_s^2}\right]. \tag{13}$$

It denotes the probability of residual nucleus with angular momentum quantum number j' in which σ_s is spin cutoff factor[10].

In the code CDCCOM, the angular momentum quantum number j' of $u_{l'j'}(r_{nA})$ is truncated if $R(j', U)/R(j'_0 = 1/2, U) \leqslant 0.01$. In this way, for a certain excited energy U, both the contribution of bound state and that of excited state of n-A system are included in the transition matrix element $T^J(\mathrm{d}, \mathrm{p})$. Therefore, formula (10) should be changed as follows

$$\begin{aligned} T^J(\mathrm{d}, \mathrm{p}) = {} & a_{J0} W^0_{\mathrm{pn}} \int_0^{R_{\max}} R\mathrm{d}R \times \\ & \sum_{i=0}^{Nk=10} \phi_0(k_i, r)\chi_J(P_{k_i}, R) \times \\ & \sum_{j'=j'_0}^{j'=j'_{\max}} R(j', U) \sum_{l'=j'\pm 1/2} u_{l'j'}(R) \times \\ & \sum_{j=|J-j'|}^{J+j'} \sum_{m_j=-j}^{j} C^{J0}_{j, m_j, j', -m_j} \sum_{l=j\pm 1/2} u_{lj}(QR) \times \\ & \sum_{\mu=\pm 1/2} a^*_{l, m_j-\mu} C^{jm_j}_{l, m_j-\mu, 1/2, \mu} C^{j, -m_j}_{l', -m_j+\mu, 1/2, -\mu} \times \\ & \sqrt{\frac{(2l+1)(2l'+1)}{(2J+1)}} C^{J0}_{l, 0, l', 0} C^{J0}_{l, m_j-\mu, l', -m_j+\mu} . \end{aligned} \tag{14}$$

With formula (14), we can get the differential cross section of stripping reaction at outgoing proton energy $E_p = E - U - B_{nA}$. Therefore, for the definite deuteron incident energy, the outgoing proton energy spectra induced by deuteron breakup effect can be calculated in whole energy range.

5 Results

For d+^{58}Ni reaction with incident energy of deuteron 80 MeV, the outgoing proton energy spectra induced by deuteron breakup effects is calculated by CDCCOM, and the calculated result is shown in Fig. 1. Our calculated result is plotted as solid line, and the dash line of Gaussian shape expresses that from C. Kalbach's empirical formula. From Fig. 1 we know that, our result calculated with CDCCOM and that of Kalbach are generally agree with each other, namely, a peak structure appears in breakup effect energy spectra at about half of the incident energy. Our calculated results prove that the peak structure exists in outgoing proton energy spectra, and Kalbach's empirical formula is of rationality.

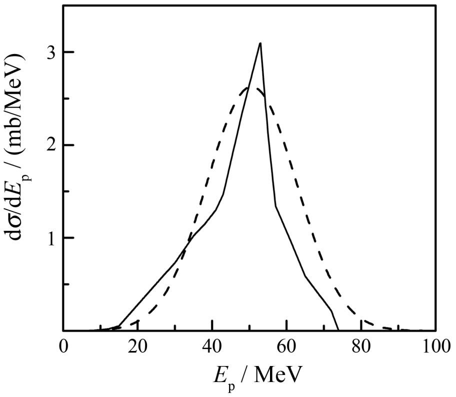

Fig. 1 The outgoing proton energy spectra induced by deuteron breakup effect in d+^{58}Ni reaction at 80 MeV. The solid line is the result of present work, and the dash line is that from C. Kalbach's empirical formula.

However, there are some differences between our result and Kalbach's. Firstly, the energy spectra calculated by Kalbach's empirical formula is symmetric, and that of ours is not symmetric, with more obvious tendency towards higher outgoing energy. Secondly, the area covered by the curve of Kalbach's is larger than that of ours, this indicates that the contribution of empirical formula is little larger than our theoretical calculation to proton energy spectra; Thirdly, the peak value of Kalbach's is lower than ours, the energy range of breakup effects of Kalbach's empirical formula is larger than that of ours.

6 Summary

Based on the hypothesis that the residual nucleus locates not only in bound state but also in a series of excited states, the three body wave function calculated by CDCC theory, and the transition matrix element of stripping reaction, the outgoing proton energy spectra induced by deuteron breakup effect in d+^{58}Ni reaction at 80 MeV were studied. The calculated result shows that a peak structure is appeared at about half of the incident energy, it physically indicates that the deuteron breakup effect in outgoing proton energy spectra, and proves the reasonability of C. Kalbach's empirical formula.

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氘核破裂效应对质子出射能谱的贡献*

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摘　要：基于连续离散化耦合道理论(CDCC)，并在假设剩余核不仅可处于束缚态而且可处于一系列的激发态的基础上，本工作论述了氘核破裂效应对质子出射能谱的贡献。计算结果可以基本解释质子出射能谱在大约氘核入射能量一半处出现峰值结构的实验现象。本工作的计算结果与 C. Kalbach 的经验公式基本保持一致。

关 键 词：氘核；破裂效应；CDCC 理论；质子出射能谱

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