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Deuteron Form Factor $A(Q^2)$ in the Paris Potential Wave Function

PENG Jinsong[1], MENG Chengju[1], PAN Jihuan[1], ZHOU Lijuan[2], MA Weixing[3]

(*1. School of Physics and Electronic Engineering, Hechi University, Yizhou 546300, Guangxi, China*;
2. College of Science, Guangxi University of Science and Technology, Liuzhou 545006, Guangxi, China;
3. Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China)

Abstract: Within non-relativistic framework and using deuteron wave function produced by the Paris potential which is designed just for describing nuclear process at high momentum transfers, we calculate deuteron electromagnetic form factor $A(Q^2)$ which is formulized as a product of the deuteron structure function $Z(t)$ and dipole form factor $G_{\mathrm{D}}(t)$. The structure function $Z(t)$ is obtained numerically from the Paris potential wave function calculations. Our theoretical prediction for $A(Q^2)$ is in a good agreement with experimental data available. This successful fit to data shows that the simple theoretical description of deuteron form factors $A(Q^2)$ works quite well, and could be applied to studying other static properties of deuteron.

Key words: electromagnetic form factor of deuteron; Paris potential wave function; elastic electron scattering on deuteron

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1 Introduction

The deuteron, as a simple spin-1 nucleus, is usually believed to be a weakly bound system of a proton and a neutron with the binding energy of $E_{\mathrm{d}}=2.22$ MeV. Since the electromagnetic properties of the deuteron shed some lights on electromagnetic form factor of the neutron which can not be measured directly, the studying of the deuteron form factors with an electromagnetic probe is of great interest. A long and rich history review on this subject can be found in Refs. [1-4].

The deuteron form factors, are usually measured directly by elastic electron scattering on deuteron, e+d→e+d. Theoretically, the operator matrix elements for elastic electron scattering off deuteron in the one-photon exchange approximation are given[5] by

$$M=\frac{e^2}{Q^2}\bar{u}_{\mathrm{e}'}(K')\gamma_\mu\mu_{\mathrm{e}}(K)J^{\mathrm{d}}_\mu(p,p') \tag{1}$$

where K and K' are the four momentum of the initial and final electron, respectively. $J^{\mathrm{d}}_\mu(p,p')$ is the deuteron electromagnetic current

$$J^{\mathrm{d}}(p,p')_\mu=-\left[G_1(Q^2)\lambda'^*\cdot\lambda-\frac{G_3(Q^2)}{2M_{\mathrm{d}}^2}\lambda\cdot q\lambda'^*\cdot q\right]\times (p+p')_\mu-G_2(Q^2)(\lambda_\mu\lambda'^*\cdot q-\lambda'^*_\mu\lambda\cdot q)\ , \tag{2}$$

where M_{d} is the deuteron mass, $\lambda(\lambda')$ and $p(p')$ are polarization and four momentum of the initial (final) deuteron with $q=p'-p$ being the momentum transfer. The three electromagnetic form factors in Eq. (2), G_1, G_2 and G_3, are related to the charge form factor G_{C}, quadruple form factor G_{Q}, and magnetic

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Biography: Peng Jinsong(1963–), male, Guangxi Hechi, Professor ,working on the field of theory physics: particle physics and nuclear physics; E-mail:123pjs@163.com
Corresponding author: MA Weixing, E-mail: mawx@ihep.ac.cn.

form factor $G_{\rm M}$ by

$$G_{\rm C}=G_1=\frac{2}{3}\tau G_{\rm Q}\ ,\quad G_{\rm M}=G_2\ ,$$
$$G_{\rm Q}=G_1-G_2+(1+\tau)G_3\ ,\ \ \tau=\frac{Q^2}{4M_{\rm d}^2}\ . \qquad (3)$$

These form factors are normalized at zero recoil momentum in such a way that

$$G_{\rm C}(0)=1\ ,\quad G_{\rm Q}(0)=M_{\rm d}^2Q_{\rm d}^2=25.83\ ,$$
$$G_{\rm M}(0)=\frac{M_{\rm d}}{m_{\rm N}}\mu_{\rm d}=1.714\ , \qquad (4)$$

with $m_{\rm N}$ being the nucleon mass, $Q_{\rm d}$ and $\mu_{\rm d}$ are the quadruple and magnetic moments of the deuteron. Since the spin of deuteron is one, it has three electromagnetic form factors in the one-photon exchange approximation. Due to current conservation, the parity P and charge conjugation C are of invariance. The elastic scattering mechanism of electron off the deuteron can be diagrammatically depicted as that in Fig. 1.

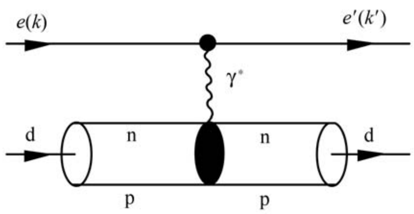

Fig. 1 Schematic representation of elastic electron scattering off the deuteron e+d→e+d, deuteron consists of a proton and a neutron.
Waving line stands for the exchange photon between incident electron and target deuteron, while the small and large dark circles denote respectively the interaction vertexes of electron with photon and photon with deuteron.

According to the Rosenbluth separation[6], the differential cross section of an unpolarized electron elastic scattering from deuteron[7] in order of $O(\alpha^2)$ reads

$$\frac{{\rm d}\sigma}{{\rm d}\Omega}=\left(\frac{{\rm d}\sigma}{{\rm d}\Omega}\right)_{\rm Mott}\times \left[A(Q^2)+B(Q^2)\tan^2\left(\frac{\theta}{2}\right)\right]\ . \qquad (5)$$

Where $A(Q^2)$ and $B(Q^2)$ are two form factors. The form factor $A(Q^2)$ is related to the form factors $G_{\rm C}$, $G_{\rm M}$ and $G_{\rm Q}$ of deuteron by

$$A(Q^2)=G_{\rm C}^2(Q^2)+\frac{2}{3}\tau G_{\rm M}^2(Q^2)+\frac{8}{9}\tau^2G_{\rm Q}^2(Q^2)\ . \qquad (6)$$

The theoretical studies of electron-deuteron elastic scattering e+d→e+d and static electromagnetic properties of deuteron have been performed in many different approaches[1−4]. In this paper, we apply our previous formulism derived for vector meson (V) photo-production off deuteron γ +d→V+d in QCD inspired eikonalized model[8] to the prediction of the deuteron form factor $A(Q^2)$.

The paper is organized as follows: In Sect. 2, we briefly introduce theory and formulae originated from our previous publications[8]. In Sect. 3, we make the numerical calculations and present our theoretical prediction for the form factor $A(Q^2)$, which enter in differential cross section ${\rm d}\sigma(t)/{\rm d}t$ of electron elastic scattering off deuteron e+d→e+d. Our conclusions are reserved for Sect. 4.

2 Theory on $A(Q^2)$ and its resulting formulism

We closely follow the treatment of Ref. [9] and work in non-relativistic framework for the deuteron form factor based on utilizing the realistic *NN* interaction of Paris potential. Concretely speaking, we use the deuteron wave function which describes fairly well the deuteron electromagnetic form factor with momentum transfer up to $-t\approx 0.9$ GeV^2. Using this wave function, the charge form factor $F_{\rm C}$, quadruple form factor $F_{\rm Q}$ and the magnetic form factor $F_{\rm M}$ are given[9] by

$$F_{\rm C}=R_{000}+R_{220}\ ,$$
$$F_{\rm Q}=R_{202}-\frac{1}{\sqrt{8}}R_{220}\ ,$$
$$F_{\rm M}=R_{000}+\frac{1}{2}R_{220}+\sqrt{2}R_{202}\ , \qquad (7)$$

where the radial integrals $R_{\rm LL'\lambda}(q)$ are given by

$$R_{\rm LL'\lambda}(q)=\int{\rm d}rU_{\rm L}(r)U'_{\rm L}(r)j_\lambda(qr) \qquad (8)$$

with $U_{\rm L}(r)$ being the radial part of deuteron wave function. Let us now define

$$Z(t)=F_{\rm C}^2(t)+F_{\rm Q}^2(t) \qquad (9)$$

which is related to the well-known structure function of electron elastic scattering on the deuteron e+d→e+d as

$$A(t)=Z(t)G_{\rm D}^2(t) \qquad (10)$$

with t being four-momentum transfer. $G_D(t)$ is the so-called dipole form factor of the nucleon and it has the form[10]

$$G_D(t)=\left(1-\frac{t}{0.71\ \mathrm{GeV}^2}\right)^{-2}. \qquad (11)$$

The relation between t and Q is given by

$$t=-2M_d\left(\sqrt{Q^2+M_d^2}-M_d\right). \qquad (12)$$

The structure function $Z(t)$ is function of t, and it is obtained by numerical calculations of Paris potential deuteron wave functions. We depict the t-dependence of $Z(t)$ in Fig. 2. From Eqs. $(10\sim 11)$, we can predict the Q^2-dependence of $A(Q^2)$. The resulting prediction is shown in Fig. 3.

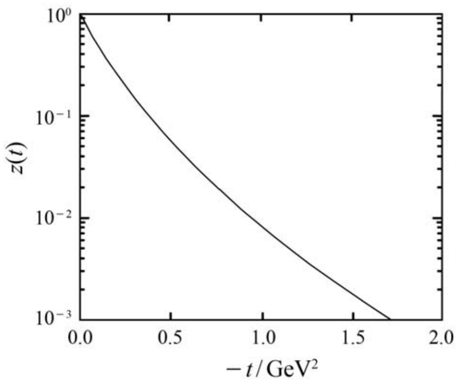

Fig. 2　t-Dependence of the structure function $Z(t)=F_C^2(t)+F_Q^2(t)$ are calculated numerically.

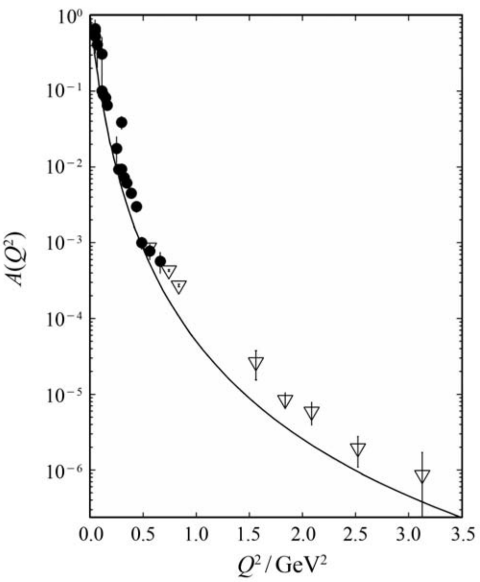

Fig. 3　Q^2-dependence of deuteron form factor $A(Q^2)$. The predictions are given by Eq. (10). The solid curve is our theoretical prediction and the data come from Refs. [11-12].

3　Numerical results and discussions

Using Eq. (10), Eq. (11) and the function $Z(t)$ given in Fig. 2, we obtain the deuteron form factors $A(Q^2)$. In Fig. 3, the experimental data are also shown for a comparison between our theoretical prediction and data. The data are quoted from Refs. [11-12].

As is seen from Fig. 3, our predictions are in good agreement with data. However, we need to go further to predict the form factor $B(Q^2)$, which is quite complicated to calculate, and then the related differential cross section $\mathrm{d}\sigma(t)/\mathrm{d}t$ to test the validity of our theory. This work will come up soon.

4　Conclusions

Based on deuteron wave function determined by the Paris potential, we study deuteron electromagnetic form factor $A(Q^2)$ under one photon exchange approximation. This wave function was believed to describe fairly well the elastic scattering of electron on deuteron up to momentum transfer $-t=0.9\ \mathrm{GeV}^2$. We obtain a good agreement between our theoretical calculations and experimental data. Therefore, our conclusion is that this simple model could explain the data and we do not need any other degrees of freedom in fitting to data at the low momentum transfers region, $-t\leqslant 0.9\ \mathrm{GeV}^2$. Of course, for higher momentum transfers, we need new degrees of freedom to put into our theoretical considerations of the deuteron structure. For instance, at high momentum transfers, the quark degrees of freedom and meson exchange contributions, such as meson exchange current, six quark cluster component of deuteron wave function, and quark-antiquark $q\bar{q}$ pair current, maybe make large contributions to the form factors. The work along this line is in progress.

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在巴黎势的波函数中氘核的形状因子$A(Q^2)$

彭金松[1]，蒙成举[1]，潘继环[1]，周丽娟[2]，马维兴[3]

（1. 河池学院物理与机电工程学院，广西 宜州 546300；
2. 广西科技大学理学院，广西 柳州 545006；
3. 中国科学院高能物理研究所，北京 100049）

摘要： 在非相对论的框架内和应用描述了高动量过程的巴黎势所产生的氘核的波函数，计算了氘核的电磁形状因子$A(Q^2)$。$A(Q^2)$因子化为氘核的结构函数$Z(t)$和偶极形状$G_{\mathrm{D}}(t)$之积。结构函数$Z(t)$是用巴黎势的波函数做数值计算而得到的。对$A(Q^2)$的预言跟实验数据的分析非常一致，这一对实验结果的成功描述说明：氘核形状因子$A(Q^2)$这一简单的理论描述是非常成功的，并且可以用到氘核的其他静态性质的研究。

关键词： 氘核的电磁形状因子；巴黎势波函数；电子在氘核上的弹性散射

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通信作者： 马维兴，E-mail: mawx@ihep.ac.cn。