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# Multiplicity Fluctuation Analysis of the Target Recoil Protons in Nucleus-emulsion Collisions at a few Hundred MeV/nucleon

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**Abstract:** Multiplicity fluctuation of the target recoil protons emitted in 290 A MeV  $^{12}\text{C}$ -AgBr, 400 A MeV  $^{12}\text{C}$ -AgBr, 400 A MeV  $^{20}\text{Ne}$ -AgBr and 500 A MeV  $^{56}\text{Fe}$ -AgBr interactions are studied using the scaled factorial moment (SFM) method in two-dimensional normal phase space and cumulative variable space, respectively. It is found that in normal phase space the SFM ( $\ln\langle F \rangle$ ) increases linearly with the increase of the divided number of phase space ( $\ln M$ ) for lower  $q$ -values and increases linearly and then becomes saturation or decrease with the increase of  $\ln M$  for higher  $q$ -values, and in cumulative variable space  $\ln\langle F \rangle$  decreases linearly with the increase of  $\ln M$ , which indicates that no evidence of non-statistical multiplicity fluctuation is observed in our data sets.

**Key words:** heavy ion collision; target recoil proton; multiplicity fluctuation; nuclear emulsion

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## 1 Introduction

Multiplicity fluctuation includes the statistical fluctuation and the non-statistical fluctuation. The statistical fluctuation or noise contaminates the non-statistical fluctuation or the dynamical fluctuation. Bialas and Peschanski<sup>[1]</sup> proposed the scaled factorial moment (SFM) method to study the non-statistical fluctuation in multi-particle production, which can disentangle the statistical noise and measure the dynamical fluctuation. The non-statistical fluctuation or intermittency is a manifestation of scale invariance of the physical process and randomness of the underlying scaling law, which is defined as the power law growth of the SFM with the decreasing phase space interval size.

In high energy nucleus-emulsion collisions, target fragmentation produces grey and black track particles. The grey track particle is formed due to the target recoil protons with the medium energy of 30 ~ 400 MeV, which is supposed to carry some information about the interaction dynamics because the time scale of the emission of these particles is of the same order

( $\approx 10^{-22}$  s) as that of the produced particles. These protons are the low energy part of the internuclear cascade formed in high energy interactions.

The non-statistical fluctuation of the emission of the target recoil protons in high energy nucleus-nucleus collisions were investigated not only in one-dimensional phase space<sup>[2-7]</sup> but also in two-dimensional phase space<sup>[8]</sup>, the evidence of dynamical fluctuation was obtained by using the SFM method or Takagi-moment method<sup>[9]</sup> in most of these investigation. In Ref. [3] it is found that the SFM grew according to power law with phase space interval size decreasing and the saturation effects was observed, and in Ref. [8] the SFM decrease according to power law with phase space interval size decreasing was also observed. These effects indicate that there is not a dynamical fluctuation in the emission of the target recoil protons. In our previous investigation<sup>[10]</sup> of the target recoil protons at a wide range of energies from 500 A MeV to 60 A GeV also revealed the same multiplicity fluctuation property. In intermediate and high energy (a few hundred MeV/nucleon) nucleus-nucleus collisions, investigation of the non-statistical fluctua-

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tion in the emission of the target recoil protons is not paid much attention.

In this paper, the non-statistical fluctuation of the target associated protons produced in 290 A MeV  $^{12}\text{C}$ -AgBr, 400 A MeV  $^{12}\text{C}$ -AgBr, 400 A MeV  $^{20}\text{Ne}$ -AgBr and 500 A MeV  $^{56}\text{Fe}$ -AgBr interactions are studied in two-dimensional phase space by using the SFM method.

## 2 Experimental details

Four stacks of nuclear emulsion made by Institute of Modern Physics, Shanxi Normal University, were used in present investigation. The emulsion stacks were exposed horizontally at Heavy Ion Medical Accelerator in Chiba (HIMAC), National Institute of Radiological Science (NIRS), Japan. The beams were 290 A MeV  $^{12}\text{C}$ , 400 A MeV  $^{12}\text{C}$ , 400 A MeV  $^{20}\text{Ne}$  and 500 A MeV  $^{56}\text{Fe}$  respectively, and the flux was about 3 000 ions/cm<sup>2</sup>. BA2000 and XSJ-2 microscopes with a 100× oil immersion objective and 10× ocular lenses were used to scan the plates. The tracks were picked up at a distance of 5 mm from the edge of the plates and were carefully followed until they either interacted with emulsion nuclei or escaped from the plates. Interactions which were within 30 μm from the top or bottom surface of the emulsion plates were not considered for final analysis. All the primary tracks were followed back to ensure that the events chosen do not include interactions from the secondary tracks of other interactions. When they were observed to do so the corresponding events were removed from the sample.

In each interaction all of the secondaries were recorded including shower particle, target recoiled proton (grey track particle), target evaporated fragment (black track particle) and projectile fragments. Shower particles are produced single-charged relativistic particles having velocity  $v \geq 0.7 c$ , the multiplicity is denoted as  $N_s$ . Grey track particles are mostly the recoiled protons with the kinetic energy of  $26 \leq E \leq 375$  MeV, the multiplicity is denoted as  $N_g$ . Black track particles are the target evaporated fragments with the kinetic energy of  $E \leq 26$  MeV, the multiplicity is denoted as  $N_b$ . Grey and black track particles together are called the heavy ionizing particles, the multiplicity is denoted as  $N_h$ . Projectile fragments are the residues of projectile with constant ionization, long range and small emission angle. Details of track classification can be found in Ref. [11].

To ensure that the targets in nuclear emulsion are silver or bromine nuclei, we have chosen only the events with at least eight heavy ionizing track particles ( $N_h \geq 8$ ).

## 3 Method of analysis

The non-statistical multiplicity fluctuation analysis is performed in a two-dimensional phase space. The phase space is divided equally in both directions assuming that is isotropic in nature. Denote the two phase space variables as  $x_1$  and  $x_2$ , then horizontal SFM of order  $q$  is defined as that in Ref. [1]

$$F_{qi}(\delta x_1 \delta x_2) = M^{q-1} \sum_{m=1}^M \frac{n_{mi}(n_{mi}-1) \cdots (n_{mi}-q+1)}{n_i(n_i-1) \cdots (n_i-q+1)}, \quad (1)$$

where  $\delta x_1 \delta x_2$  is the size of a two-dimensional cell,  $n_{mi}$  is the multiplicity in the  $m$ -th cell of the  $i$ -th event,  $n_i$  is the multiplicity of the  $i$ -th event, and  $M$  is the number of two-dimensional cells into which the considered phase space has been divided. Then the averaged horizontal SFM becomes

$$\langle F_q(\delta x_1 \delta x_2) \rangle = \frac{1}{N_{\text{ev}}} \sum_{i=1}^{N_{\text{ev}}} F_{qi}(\delta x_1 \delta x_2). \quad (2)$$

Non-statistical multiplicity fluctuation would manifest itself as a power-law scaling of  $\langle F_q \rangle$  with the cell size of the form

$$\langle F_q \rangle \propto (\delta x_1 \delta x_2)^{-a_q} \quad \text{as } \delta x_1 \delta x_2 \rightarrow 0, \quad (3)$$

or a linear relation

$$\ln \langle F_q \rangle = -a_q \ln(\delta x_1 \delta x_2) + b_q = a_q \ln M + c_q. \quad (4)$$

The invariant quantity of the scaling  $a_q > 0$  is called the intermittency exponent, and it is a measure of the fluctuation strength.

The single-particle density distribution in two-dimensional space (emission angle space and azimuthal angle space) is non-flat. As the shape of this distribution influences the scaling behavior of the SFMs. Bialas and Gazdzicki<sup>[12]</sup> introduced ‘‘cumulative’’ variable which drastically reduced the distortion of intermittency due to non-uniformity of single-particle density distribution. Following them, the cumulative variable  $X(x)$  is related to the single-particle density distribution  $\rho(x)$  through

$$X(x) = \frac{\int_{x_1}^x \rho(x') dx'}{\int_{x_1}^{x_2} \rho(x') dx'}, \quad (5)$$

where  $x_1$  and  $x_2$  are two extreme points of the distribution  $\rho(x)$ . The variable  $X(x)$  varies between 0.0 and 1.0, with  $\rho(X(x))$  kept almost constant. The values of  $x_1$  and  $x_2$  are  $-1$  and  $1$  in  $\cos\theta$ -space,  $0$  and  $2\pi$  in azimuthal angle space, respectively.

## 4 Results and discussion

Figs. 1 and 2 show the emission angle and azimuthal angle distributions of the target recoil protons emitted from 290 AMeV  $^{12}\text{C}$ -AgBr, 400 AMeV  $^{12}\text{C}$ -AgBr, 400 AMeV  $^{20}\text{Ne}$ -AgBr and 500 AMeV  $^{56}\text{Fe}$ -AgBr interactions. It is found that the angular distributions are not uniform in whole phase space. Most of the target recoil protons are emitted in forward hemisphere (emission angle less than  $90^\circ$ ). The azimuthal angle distributions of the target recoil protons are approximately symmetric around  $\phi = 180^\circ$ . It is natural that the azimuthal angle distribution of produced particles in intermediate and high energy nucleus-nucleus collisions should be symmetric around  $\phi = 180^\circ$ , especially for target fragments.

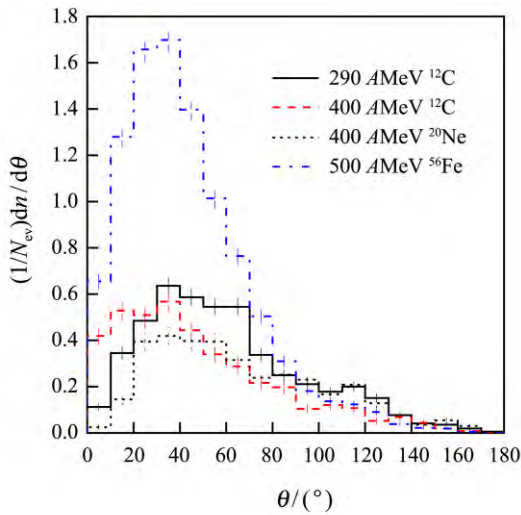


Fig. 1 (color online) Emission angle distribution of the target recoil protons.

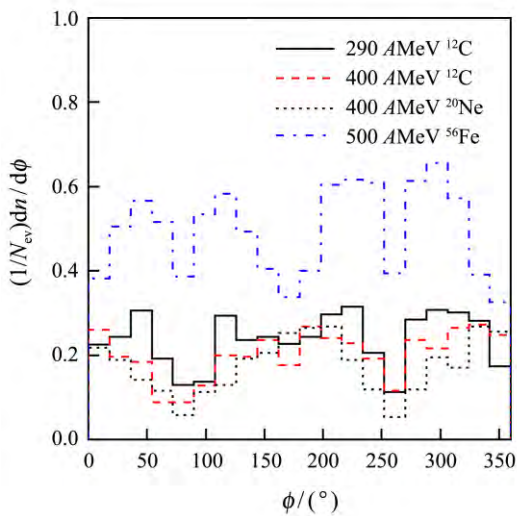


Fig. 2 (color online) Azimuthal angle distribution of the target recoil protons.

The distribution of grey track particles peaks forward in emission angle and therefore they are “closer” to the primary sequence of primary nucleon-nucleon collisions. One generally assumes that the grey track particles originate from the intranuclear cascade which is initiated by the recoiling primary nucleons. The observed grey track particles are partly “primary” protons, *i.e.* struck by the incident hadron, or “secondary” protons, *i.e.* knocked-out of the nucleus by the primary nucleons. The incident nucleons in  $^{56}\text{Fe}$ -AgBr collision system are almost three times than ones in  $^{20}\text{Ne}$ -AgBr and  $^{12}\text{C}$ -AgBr collision systems, so the production of grey track particles in  $^{56}\text{Fe}$ -AgBr collision system are greater than that in  $^{20}\text{Ne}$ -AgBr and  $^{12}\text{C}$ -AgBr collision systems. From Fig. 1 and Fig. 2 we can clearly see that the distribution for  $^{56}\text{Fe}$ -AgBr collision is different from that for  $^{20}\text{Ne}$ -AgBr and  $^{12}\text{C}$ -AgBr collisions.

Fig. 3 shows the dependence of  $\ln\langle F_q \rangle$  on  $\ln M$  for the target recoil protons emitted from 290 AMeV  $^{12}\text{C}$ -AgBr, 400 AMeV  $^{12}\text{C}$ -AgBr, 400 AMeV  $^{20}\text{Ne}$ -AgBr and 500 AMeV  $^{56}\text{Fe}$ -AgBr interactions in normal variable space. It can be seen that for lower order  $q$  the  $\ln\langle F_q \rangle$  increases linearly as  $\ln M$  increases, but for higher order  $q$  the  $\ln\langle F_q \rangle$  increases linearly with an increase of  $\ln M$  and then becomes saturation or decrease. All of the distribution is fitted based on Eq. (4). The fitting parameters  $a_q$  and  $c_q$  including the minimum  $\chi^2/\text{DOF}$  are presented in Table 1, where DOF means the degree of freedom of simulation. The fitted intermittency exponents  $a_q > 0$  for all of the interactions, and  $a_q$  increases with the increase of  $q$ . We cannot get a clear evidence of the non-statistical fluctuation from the results of Fig. 3.

Fig. 4 shows the dependence of  $\ln\langle F_q \rangle$  on  $\ln M$  for the target recoil protons emitted from 290 AMeV  $^{12}\text{C}$ -AgBr, 400 AMeV  $^{12}\text{C}$ -AgBr, 400 AMeV  $^{20}\text{Ne}$ -AgBr and 500 AMeV  $^{56}\text{Fe}$ -AgBr interactions in cumulative variable space. The distribution is fitted using Eq. (4). The fitting parameters  $a_q$  and  $c_q$  including the minimum  $\chi^2/\text{DOF}$  are presented in Table 1. The fitted intermittency exponents  $a_q < 0$  for all of the interactions, and  $a_q$  decreases with the increase of  $q$ . It can be seen that  $\ln\langle F_q \rangle$  decreases linearly with  $\ln M$  increases, which indicate that no evidence of non-statistical multiplicity fluctuation is observed in our data sets. So any fluctuation indicated in Fig. 3 is totally caused by non-uniformity of single-particle density distribution.

It is known that a useful measure of the fluctuation of any variable is the ratio of its variance to its mean<sup>[13]</sup>. So the variance of the target recoil proton multiplicity distribution can be used to measure the target associated proton multiplicity fluctuation. In

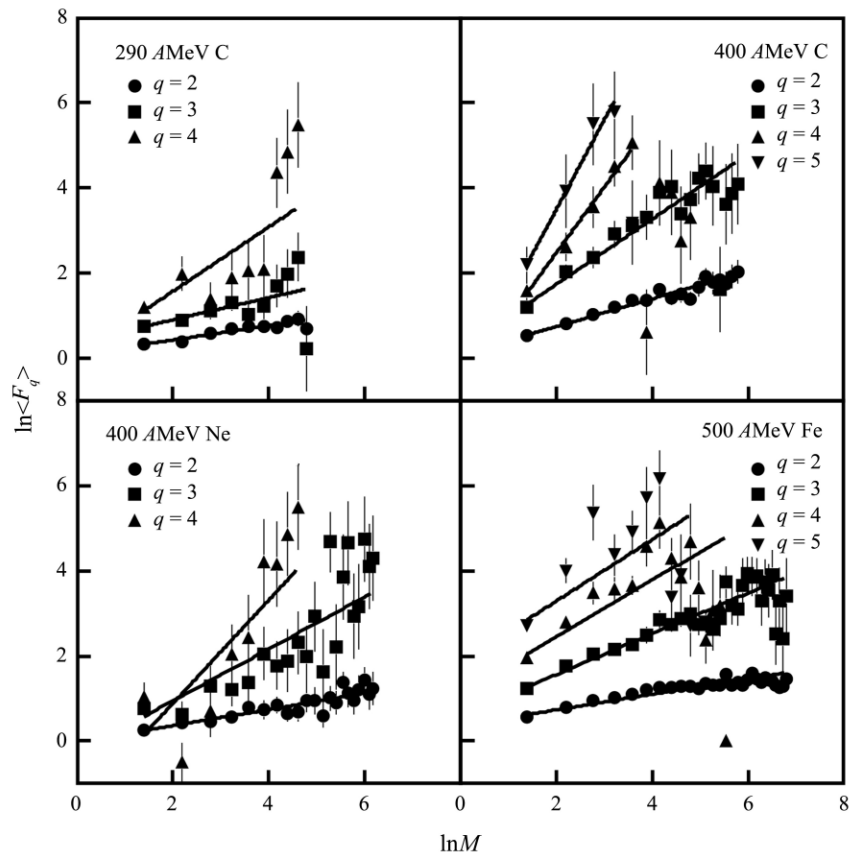


Fig. 3 Plots of  $\ln\langle F_q \rangle$  against  $\ln M$  in two-dimensional normal variable space.

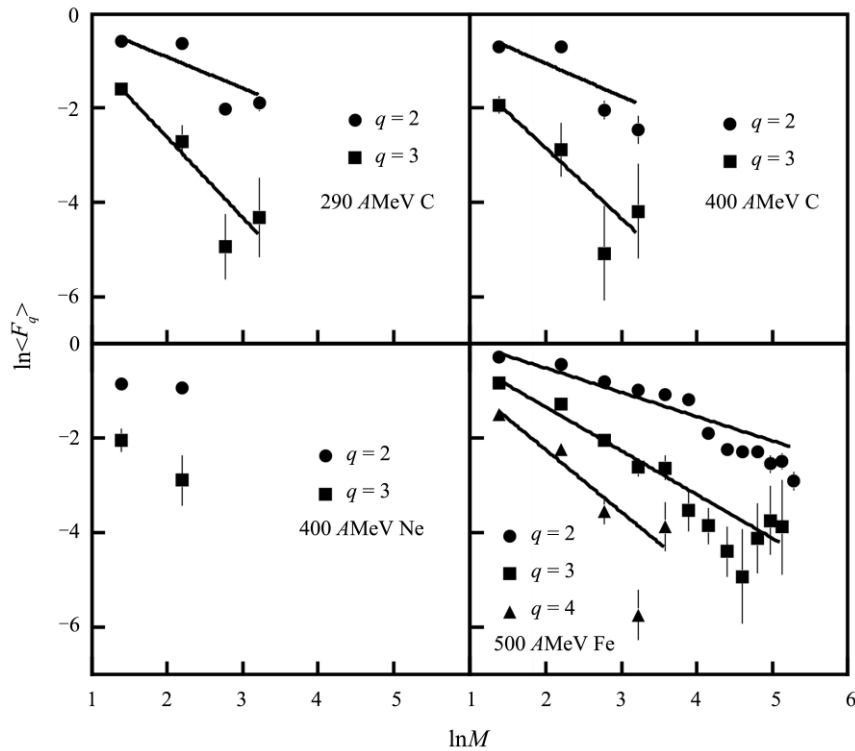


Fig. 4 Plots of  $\ln\langle F_q \rangle$  against  $\ln M$  in two-dimensional cumulative variable space.

Table 1 The best fitting parameters of correlation  $\ln\langle F_q \rangle$  and  $\ln M$  using Eq. (4) for heavy ion induced AgBr targets interactions.

Interaction	$q$	Normal variable space			Cumulative variable space		
		$a_q$	$c_q$	$\chi^2/\text{DOF}$	$a_q$	$c_q$	$\chi^2/\text{DOF}$
290 AMeV $^{12}\text{C}+\text{AgBr}$	2	$0.169\pm 0.023$	$0.086\pm 0.059$	0.432	$-0.657\pm 0.064$	$0.397\pm 0.121$	27.78
	3	$0.278\pm 0.067$	$0.345\pm 0.157$	0.626	$-1.699\pm 0.279$	$0.776\pm 0.446$	1.49
	4	$0.756\pm 0.140$	$0.059\pm 0.319$	2.267			
400 AMeV $^{12}\text{C}+\text{AgBr}$	2	$0.327\pm 0.022$	$0.086\pm 0.067$	0.447	$-0.670\pm 0.103$	$0.348\pm 0.193$	13.68
	3	$0.858\pm 0.073$	$0.031\pm 0.179$	0.230	$-1.492\pm 0.399$	$0.140\pm 0.647$	0.80
	4	$1.543\pm 0.210$	$-0.604\pm 0.457$	0.171			
400 AMeV $^{20}\text{Ne}+\text{AgBr}$	2	$0.190\pm 0.025$	$-0.018\pm 0.082$	0.406			
	3	$0.609\pm 0.068$	$-0.270\pm 0.214$	1.179			
	4	$1.213\pm 0.197$	$-1.535\pm 0.532$	3.909			
500 AMeV $^{56}\text{Fe}+\text{AgBr}$	2	$0.183\pm 0.006$	$0.372\pm 0.020$	2.504	$-0.514\pm 0.015$	$0.511\pm 0.035$	13.21
	3	$0.479\pm 0.020$	$0.609\pm 0.057$	0.929	$-0.929\pm 0.051$	$0.516\pm 0.105$	1.76
	4	$0.663\pm 0.057$	$1.136\pm 0.131$	3.381	$-1.319\pm 0.127$	$0.388\pm 0.240$	6.74
	5	$0.722\pm 0.104$	$1.853\pm 0.228$	4.633			

order to study the target recoil proton multiplicity fluctuation in nucleus-nucleus collisions, we define a scaled variable  $w$  such that

$$w = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle}. \quad (6)$$

A direct measure of the scaled variance would give a direct measure of multiplicity fluctuation. Multiplicity fluctuation is but one aspect of a two-particle correlation function. The study of the scaled variance can very easily reveal the nature of correlation among the produced particles. If the value of  $w$  is much greater than 1, it may be said that there is a strong correlation among the produced particles. In contrast, if the value of  $w$  is close to one, weak correlation is indicated.

The values of the scaled variance  $w$  for all of the interactions were calculated in order to quantify the correlations among the target recoil protons emitted from 290 AMeV  $^{12}\text{C}-\text{AgBr}$ , 400 AMeV  $^{12}\text{C}-\text{AgBr}$ , 400 AMeV  $^{20}\text{Ne}-\text{AgBr}$  and 500 AMeV  $^{56}\text{Fe}-\text{AgBr}$  interactions. It is found that the values of the scaled variance are  $1.59\pm 0.41$ ,  $1.34\pm 0.44$ ,  $1.61\pm 0.47$  and  $3.86\pm 0.91$  respectively for the production of the target recoil protons in 290 AMeV  $^{12}\text{C}-\text{AgBr}$ , 400 AMeV  $^{12}\text{C}-\text{AgBr}$ , 400 AMeV  $^{20}\text{Ne}-\text{AgBr}$  and 500 AMeV  $^{56}\text{Fe}-\text{AgBr}$  interactions, which suggested that there is a weak correlation in production of the target recoil protons.

## 5 Conclusions

The non-statistical multiplicity fluctuation of the target recoil protons emitted from 290 AMeV  $^{12}\text{C}-\text{AgBr}$ , 400 AMeV  $^{12}\text{C}-\text{AgBr}$ , 400 AMeV  $^{20}\text{Ne}-\text{AgBr}$

and 500 AMeV  $^{56}\text{Fe}-\text{AgBr}$  interactions are investigated in the framework of two-dimensional SFM methodology not only in normal phase space but also in cumulative variable space, no evidence of dynamical fluctuation is found in our data sets. Analysis of the scaled variance further revealed that there is not a strong correlation among the production of the target recoil protons in present investigation.

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## 几百兆电子伏特/核子原子核诱发乳胶核反应靶核反冲质子多重数涨落分析

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**摘要:** 利用标度阶乘矩方法对 290 A MeV  $^{12}\text{C}$ -AgBr, 400 A MeV  $^{12}\text{C}$ -AgBr, 400 A MeV  $^{20}\text{Ne}$ -AgBr 及 500 A MeV  $^{56}\text{Fe}$ -AgBr 作用靶核反冲质子在二维正常相空间及累积变量空间发射过程中的多重数涨落分别进行了分析。实验结果表明: 在正常相空间, 对于秩数  $q$  较小时标度阶乘矩 ( $\ln\langle F \rangle$ ,  $F$  为标度阶乘矩) 随相空间的分割数 ( $M$ ) 的增加而增加, 而对于秩数较大时标度阶乘矩 ( $\ln\langle F \rangle$ ) 随相空间的分割数的增加表现出先增加后趋于饱和或减小的趋势; 在累积变量空间, 标度阶乘矩 ( $\ln\langle F \rangle$ ) 随相空间的分割数的增加而减小, 这表明对于我们所研究的核作用体系靶核反冲质子发射过程中不存在非统计涨落。

**关键词:** 重离子碰撞; 靶核反冲质子; 多重数涨落; 原子核乳胶

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