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Testing CVC and CKM Unitarity via Superallowed Nuclear Beta Decay

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Abstract: Superallowed nuclear beta decay between 0^+ analog states is a sensitive probe of the weak interaction, with the established strength – or $\mathcal{F}t$ value – of each such transition being a direct measure of the vector coupling constant, G_V. Each transition's $\mathcal{F}t$ value depends on the half-life of the parent nucleus as well as on the Q-value and branching ratio for the transition of interest. It also depends on small (~1%) transition-dependent theoretical corrections, of which the most sensitive accounts for isospin symmetry breaking. We have recently published a new survey of world superallowed-decay data, which establishes the $\mathcal{F}t$ values of 14 separate superallowed transitions to a precision of order 0.1% or better. The results from this very robust data set yield the value of V_{ud}, the up-down quark mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and lead to the most demanding test available of CKM unitarity. The survey results and their outcome are described, as is the current direction of experiments that focus on testing the validity of the isospin-symmetry-breaking corrections.

Key words: beta decay; CKM unitarity; isospin symmetry breaking

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1 Introduction

Superallowed beta decay between nuclear analog states with $T = 1$ and $J^\pi = 0^+$ has been a subject of continuous and often intense study for six decades. This is because it affords a clear view of the vector current of the weak interaction, isolated from the axial-vector current and almost unobstructed by the ambiguities of nuclear structure. Angular momentum conservation completely rules out the axial-vector current, which must carry off unit spin and cannot connect two states that both have spin zero. Furthermore, since the parent and daughter states are analogs of one another, the strength of the transition is affected only by the small difference between the parent and daughter configurations resulting from isospin symmetry breaking, not by the dominant nuclear structure common to them both.

The measured strength of any β transition can be expressed as the product of the phase-space factor, f, and the partial half-life of the transition, t. This ft value depends on three measured quantities: the total transition energy, Q_{EC}, the half-life, $t_{1/2}$, of the parent state, and the branching ratio, R, for the particular transition of interest. The Q_{EC} value is required to determine f, while the half-life and branching ratio combine to yield the partial half-life.

For $0^+ \rightarrow 0^+$ transitions, the measured ft value can then be related directly to the vector coupling constant, G_V, with the intervention of only a few small (~1%) calculated terms to account for radiative and isospin symmetry-breaking effects. Once G_V has been determined in this way, it is only another short step to obtain a value for V_{ud}, the up-down mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, with which it is possible to test the top-row unitarity of that matrix. Since the unitary CKM matrix is a central pillar of the three-generation Standard Model, any experimentally determined deviation from CKM unitarity would be a signature of new physics beyond the Model; and even uncertainty limits on a sum that agrees with unitarity can serve as a constraint on possible candidates for new physics.

The broad impact of these nuclear-physics measurements has attracted considerable experimental and theoretical attention, with many groups worldwide having contributed measurement results as well as calculations of radiative and isospin-symmetry-breaking

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effects. Currently, the measurements have achieved a very high level of precision – in the best cases, ±0.015% on the ft value – and collectively their contribution to the uncertainty on $|V_{\rm ud}|^2$ is slightly less than ±0.01%. In fact, experiment has outstripped theory, which is responsible for about ±0.04% uncertainty on $V_{\rm ud}^2$. As a result, the focus of experiments has shifted from improving experimental precision overall to the measurement of particular pairs of transitions that can potentially test and improve some of the calculated correction terms.

In what follows, we shall describe the current status of world data and outline a path that could lead to further improvements.

2 World data surveyed

In dealing with superallowed decays, it is convenient to combine some of the small correction terms with the measured ft value and define a "corrected" $\mathcal{F}t$ value. Thus, we write[1]

$$\mathcal{F}t \equiv ft(1+\delta'_{\rm R})(1+\delta_{\rm NS}-\delta_{\rm C}) = \frac{K}{2G_{\rm V}^2(1+\Delta_{\rm R}^{\rm V})}\ , \quad (1)$$

where $K = 8\,120.2787(11)\times10^{-10}$ GeV^{-4}s; $\delta_{\rm C}$ is the isospin-symmetry-breaking correction and $\Delta_{\rm R}^{\rm V}$ is the transition-independent part of the radiative correction. The terms $\delta'_{\rm R}$ and $\delta_{\rm NS}$ comprise the transition-dependent part of the radiative correction, the former being a function only of the electron's energy and the Z of the daughter nucleus, while the latter, like $\delta_{\rm C}$, depends in its evaluation on the details of nuclear structure. From this equation, it can be seen that a measurement of any one superallowed transition establishes an individual value for $G_{\rm V}$. A measurement of several of them tests the Conserved Vector Current (CVC) hypothesis that $G_{\rm V}$ is not renormalized in the nuclear medium. If $G_{\rm V}$ turns out to be constant – *i.e.* all the $\mathcal{F}t$ values are the same – then an average value for $G_{\rm V}$ can be determined and $V_{\rm ud}$ obtained from the relation $V_{\rm ud}=G_{\rm V}/G_{\rm F}$, where $G_{\rm F}$ is the well known[2–3] weak-interaction constant for purely leptonic muon decay.

It is important to note that if, instead, the $\mathcal{F}t$ values show a significantly non-statistical inconsistency, one to the other, then the remaining steps cannot be taken since inconsistency would demonstrate that the correction terms were not correct or, less likely, that CVC had been violated. Without consistency, there is no coupling "constant" and there can be no justification for extracting a value for $V_{\rm ud}$.

Early in 2015, we published[1] a new critical survey of all half-life, decay-energy and branching-ratio measurements related to 20 superallowed $0^+\to0^+$ β decays. Included were 222 individual measurements of comparable precision obtained from 177 published references. We obtained world-average ft values for each of the 18 transitions that had a complete set of data, and then applied radiative and isospin-symmetry-breaking corrections to extract corrected $\mathcal{F}t$ values. A total of 14 of these $\mathcal{F}t$ values have a precision of order 0.1% or better; their uncorrected ft values and corrected $\mathcal{F}t$ values are shown in Fig. 1.

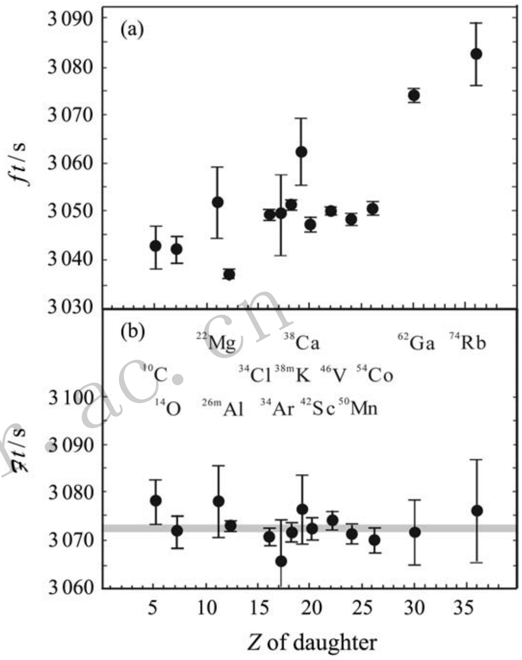

Fig. 1 Results from the 2015 survey [1]: uncorrected ft values for the 14 best known superallowed decays on the left; the same results but incorporating the $\delta'_{\rm R}$, $\delta_{\rm C}$ and $\delta_{\rm NS}$ correction terms on the right. The grey band in the right panel is the average $\mathcal{F}t$ value and its uncertainty.

It is immediately evident from the figure that the $\mathcal{F}t$ values are all consistent with one another from $A=10$ to 74. This simultaneously confirms the CVC expectation of a constant value for $G_{\rm V}$ and demonstrates the absence of any significant scalar current, which would introduce an upward or downward curve into the $\mathcal{F}t$-value locus at low Z[1]. It also goes a long way towards validating the particular set of calculated transition-dependent corrections that were used in the analysis. These calculations of $\delta_{\rm C}$ and $\delta_{\rm NS}$ were an updated version of those presented in Ref. [4] and employed the best available shell-model wave functions, which in each case had been based on a wide range of spectroscopic data for nuclei in the same mass region. They were further tuned to agree with measured binding energies, charge radii and coefficients of the isobaric multiplet mass equation for the specific states involved. This means that the origins of these correc-

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tion terms are completely independent of the superallowed decay data, so consistency in the corrected $\mathcal{F}t$ values gives powerful support to the calculated corrections used in the derivation of those $\mathcal{F}t$ values. We will return later to the question of alternative calculations for these correction terms.

With a mutually consistent set of $\mathcal{F}t$ values, one is then justified in proceeding to determine the value of G_V and, from it, V_{ud}. The result we obtained from the new survey is $V_{ud} = 0.97417(21)$ which, when combined with Particle Data Group (PDG) values for V_{us} and V_{ub}[2], yields a CKM unitarity sum of 0.99978(55), a result with 0.06% precision that is in excellent agreement with Standard Model expectations. We remark in this context though that, since the PDG evaluation was made, an inconsistency has arisen between two values of V_{us} derived from different kaon-decay modes[1]. This problem remains to be resolved.

It is instructive to look at the complete uncertainty budget for $|V_{ud}|^2$ in Fig. 2, where the four major contributions are displayed in units of parts in 10^4. By far the largest is from Δ_R^V, the so-called "inner" radiative correction. If any real improvement in the unitarity test from $0^+ \to 0^+ \beta$ decay is to be achieved in future, it must come first from improved calculations of Δ_R^V. Of course Δ_R^V is common to all transitions – and to the equivalent analysis of neutron decay for that matter – so nuclear experiments can make no contribution to reducing its uncertainty: it must remain a purely theoretical problem. However, experiment can contribute in an important way to refining the nuclear-structure-dependent corrections, δ_C and δ_{NS}, which are the second largest contributors to the $|V_{ud}|^2$ uncertainty. Since these terms exhibit very pronounced differences from transition to transition (compare the two panels in Fig. 1, which differ principally by the application of these terms) their veracity can be tested and possibly improved by new measurements that improve the measurements of inter-transitional variations, particularly those between mirror transitions.

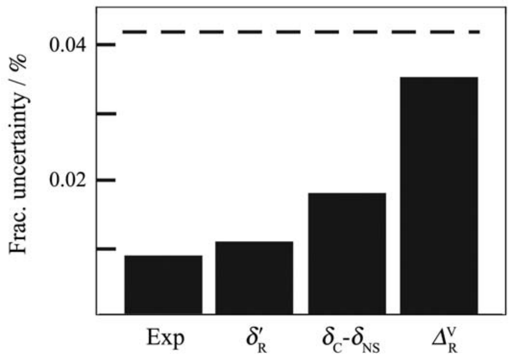

Fig. 2 Uncertainty budget for $|V_{ud}|^2$ as obtained from superallowed $0^+ \to 0^+$ β decay [1]. The contributions are separated into four categories: experiment, the transition-dependent part of the radiative correction (δ'_R), the nuclear-structure-dependent terms ($\delta_C - \delta_{NS}$) and the transition-independent part of the radiative correction Δ_R^V.

3 Tests of isospin-symmetry-breaking corrections

In the past few years, a number of different groups have published δ_C values from calculations based upon a variety of different model approaches. Typically each calculation covers only a subset of the measured transitions but the subsets are not the same from calculation to calculation and, where overlap does exist, the results are not notably consistent with one another. This diversity of results has prompted us to develop a test[5] to assess the quality of each calculated set of corrections and determine its relative merit. The test is based on the premise that the CVC hypothesis is valid and thus the corrected $\mathcal{F}t$ values for all measured transitions should be statistically consistent with one another (*i.e.* with $\chi^2/N{\sim}1$). As part of our survey, we applied this test to all sets that cover at least half the number of well-measured superallowed transitions. The resultant χ^2/N values for the various calculations spanned a wide range, with only a single set yielding a value near one. In this way, we identified that set[4], denoted SM-WS, as the one to use in our ultimate analysis of the experimental data (see Fig. 1).

There is a second test that can be expected to refine the selection process for δ_C calculations even further. It involves the measurement of mirror pairs of superallowed transitions, which has only just become possible with the first case — $^{38}Ca \to {}^{38m}K$ and $^{38m}K \to {}^{38}Ar$ — appearing very recently[6–7]. This test also depends on the expected constancy of $\mathcal{F}t$ values, but in this instance it applies to the two members of a mirror pair of $0^+ \to 0^+$ transitions. Assuming these two $\mathcal{F}t$ values are the same, we can use Eq. (1) to write the ratio of experimental ft values for a pair of mirror superallowed transitions as follows:

$$\frac{ft^a}{ft^b} = 1 + (\delta_R'^b - \delta_R'^a) + (\delta_{NS}^b - \delta_{NS}^a) - (\delta_C^b - \delta_C^a) , \quad (2)$$

where superscript "a" denotes the decay of the $T_Z = -1$ parent ($^{38}Ca \to {}^{38m}K$ in the case already referred to) and "b" denotes the decay of the $T_Z = 0$ parent ($^{38m}K \to {}^{38}Ar$). We illustrate the application of this test to two particular sets of δ_C calculations[5]. The first, identified by SM-WS, is the set that passed the χ^2 test already described, while the second, SM-HF, was used in our previous survey to establish a systematic uncertainty. The advantage offered by Eq. (2) is

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that in these models the (theoretical) uncertainty on a difference term such as $(\delta_C^b - \delta_C^a)$ is significantly less than the uncertainties on δ_C^b and δ_C^a individually.

To understand this, one must first recognize how δ_C and its quoted uncertainty were derived in the first place[4]. The term itself was broken down into two components, δ_{C1} and δ_{C2}, with the first corresponding to a finite-sized shell-model (SM) calculation typically restricted to one major shell, while the second took account of configurations outside that model space via a calculation of the mismatch between the parent and daughter radial wave functions. The parameters used for the shell-model calculation were taken from the literature, where they had been based on a wide range of independent spectroscopic data from nearby nuclei. In all cases, more than one parameter set was available, so more than one calculated value was obtained for each correction term. The value adopted for δ_{C1} was then the average of the results obtained from the different parameter sets, and the quoted "statistical" uncertainty reflected the scatter in those results. If the same approach is used to derive the mirror differences of correction terms $(\delta_{C1}^b - \delta_{C1}^a)$, the scatter among the results from different parameter sets is less than the scatter in either δ_{C1}^b or δ_{C1}^a.

For δ_{C2} there is a further source of theoretical uncertainty that arises from the choice of potential used to obtain the parent and daughter radial wave functions. Both Woods-Saxon (WS) and Hartree-Fock (HF) eigenfunctions have been used but there is a consistent difference between their δ_{C2} results[1, 5]. Although the HF version of the calculated correction terms did quite poorly on the χ^2 test for $\mathcal{F}t$ consistency, its systematic difference from the WS version had previously been viewed with some concern, as a result of which a "systematic" uncertainty was assigned to δ_{C2}. Naturally this increased the uncertainty on the derived V_{ud} and on the unitarity sum.

With the statistical (theoretical) uncertainty contribution from δ_C reduced in the mirror ft-value ratio, Eq. (2) offers the opportunity to use experiment to distinguish definitively between WS and HF radial wave functions. Considering current capabilities for producing superallowed $T_Z = -1$ parent nuclei in sufficient quantity for a high-statistics measurement, we conclude that there are three mirror pairs in addition to the one at $A = 38$ that can be completed in the immediate future. These are $^{26}Si \rightarrow {}^{26m}Al$ and $^{26m}Al \rightarrow {}^{26}Mg$; $^{34}Ar \rightarrow {}^{34}Cl$ and $^{34}Cl \rightarrow {}^{34}S$; and $^{42}Ti \rightarrow {}^{42}Sc$ and $^{42}Sc \rightarrow {}^{42}Ca$. The calculated values of ft^a/ft^b for all four are plotted in Fig. 3, along with the measured result for the $A = 38$ pair. It can be seen that the the two calculated results are cleanly separated from one another and, while the actual differences between them are small, they are large enough for experiment to be capable of selecting one calculation over the other.

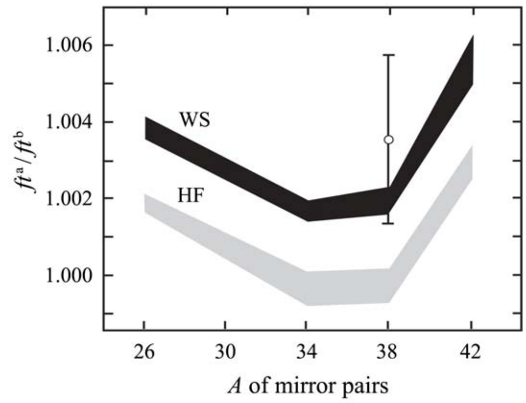

Fig. 3 Mirror-pair ft^a/ft^b values for $A = 26$, 34, 38 and 42, the four cases currently accessible to high-precision experiment. The black and grey bands connect calculated results that utilize Woods-Saxon (WS) and Hartree-Fock (HF) radial wave functions, respectively. The measured result[6–7] for the $A = 38$ mirror pair is shown as the open circle with error bars.

Although the experimental result favors the WS calculation, it is not yet definitive. The final verdict must await measurements of the other three mirror pairs, especially the one with $A = 34$. However, already the $A = 38$ result serves to confirm the outcome of the χ^2 test for $\mathcal{F}t$ consistency, a test from which only the SM-WS calculated correction terms emerged as being fully successful.

4 Future directions

A glance at Fig. 2 reveals that improvements in Δ_R^V have to be the highest priority goal for the future. The impact of any improvement would be immediate: If the Δ_R^V uncertainty were cut in half, the $|V_{ud}|^2$ uncertainty would be reduced by 30%. As already noted, though, this is a purely theoretical challenge to which nuclear experiments can contribute nothing.

Nuclear experiments can contribute, however, to improving the nuclear-structure-dependent corrections $(\delta_C - \delta_{NS})$, which produce the second largest component of the overall uncertainty budget for $|V_{ud}|^2$. The χ^2 test for $\mathcal{F}t$ consistency is a powerful discriminator among competing calculations. Obviously this test can be improved by even more precise measurements of ft values, particularly those for transitions with very large, or very small, calculated correction terms. The measured difference between two extreme cases, especially if they both involve nuclei that can be described within the same shell-model space, offers a demanding

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test. The 0.4% difference between the $^{26\mathrm{m}}$Al and ^{34}Cl ft values is a good example among the well-measured cases illustrated in Fig. 1; but $^{26\mathrm{m}}$Al and ^{30}S are expected to have an even bigger difference between them. Unfortunately this expectation cannot be tested as of now because the decay of ^{30}S has not yet been measured with useful precision. It is entirely possible that new measurements —— on the decay of ^{30}S or on other so-far uncharacterized $T_Z = -1$ nuclei —— will require and motivate further improvements to the correction-term calculations.

Completion of the mirror pairs of transitions also requires new or improved measurements of $T_Z = -1$ cases. We have already mentioned ^{26}Si, ^{34}Ar and ^{42}Ti as being important in the context of Fig. 3, but there are other cases with $A > 42$ that may soon become amenable to high-precision studies. In fact, a start on three of these cases – the decays of ^{46}Cr, ^{50}Fe and ^{54}Ni – has already been made and reported[8].

There are many more superallowed $0^+ \rightarrow 0^+$ transitions possible from $T_Z = -1$ and $T_Z = 0$ nuclei in the range $62 \leqslant A \leqslant 98$. Indeed the decays of ^{62}Ga and ^{74}Rb are both already among the well known cases illustrated in Fig. 1, with the ft value for ^{62}Ga being one of the most precisely known of them all. Unfortunately, even precisely measured ft values for these transitions cannot translate into precise $\mathcal{F}t$ values because so little is known in detail about nuclear spectroscopy in this mass region that there are no reliable nuclear-structure models upon which δ_C and δ_NS calculations can be based. It would certainly be valuable to extend the known superallowed emitters over this wider mass range, up to $A = 98$, but to be able to profit from the measurements, a great deal of spectroscopic work, both experimental and theoretical, must be completed first. Not least among the preliminary experimental tasks is to measure the masses of the three members of each of the $T = 1$ isobaric triplets that comprise a pair of superallowed emitters, and to measure the charge radii of the emitters themselves or at least some nearby isotopes. The b and c coefficients of the isobaric multiplet mass equation are used to constrain the calculation of δ_{C1}, while the charge radius is used in the calculation of δ_{C2}.

After decades of refinements, superallowed nuclear β decay has reached a remarkable level of precision in the determination of V_{ud}. Further improvements are achievable but experimentally they will be incremental and difficult to accomplish. Only theory has the potential to yield a significant gain at a single stroke: by reducing the uncertainty on the calculation of $\Delta_\mathrm{R}^\mathrm{V}$.

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