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Theoretical Study of the Coupled-channel Effects in Fusion Reactions ^{46,50}Ti+^{124}Sn

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Abstract: The heavy-ion capture and fusion processes at energies near the Coulomb barrier can be treated as a multi-dimensional barrier penetration problem. In the eigenchannel framework, the couplings to other channels split the single potential barrier into a set of discrete barriers. Based on the concept of the barrier distribution, we have developed an empirical coupled-channel (ECC) model and performed a systematic study of capture excitation functions for 220 reaction systems. Recently, an experiment was reported in which the capture excitation functions of reactions ^{46,50}Ti+^{124}Sn were measured. In this work, we review the ECC model briefly and use this model together with the universal fusion function (UFF) prescription to study the coupled-channel effects in fusion reactions ^{46,50}Ti+^{124}Sn. The reduced fusion functions show that the sub-barrier capture cross sections of ^{46}Ti+^{124}Sn exhibit an extra enhancement as compared with those of ^{50}Ti+^{124}Sn. The results from the ECC model reproduce the experimental capture excitation functions successfully and show that this extra enhancement of the sub-barrier cross sections for ^{46}Ti+^{124}Sn can be ascribed to the positive Q value neutron transfer effect

Key words: barrier distribution; empirical coupled-channel model; coupled-channel effect

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1 Introduction

During the past decades, many impressive progresses for the synthesis of superheavy nuclei have been made in both experimental and theoretical studies[1–14]. The whole process of the synthesis of superheavy nuclei can be divided into three stages: (i) the capture process in which the projectile is captured by the target and then a composite system is formed; (ii) the process of the formation of a compound nucleus, which competes against the quasifission; (iii) the deexcitation process in which the excited compound nucleus cools down through emitting neutrons and γ rays. The experimental evaporation-residue (ER) cross sections are nicely described by most of the theoretical models within less than one order of magnitude[3, 12, 14]. However, the calculated probabilities of formation of compound nucleus can differ by two or three orders of magnitude because of some serious ambiguities in the mechanism of fusion dynamics[3, 12, 14]. This means that the capture and deexcitation steps can accommodate the large discrepancies between the calculated formation probabilities and thus can reproduce the measured ER data properly. In addition, the discrepancies of the predictions for the ER cross sections for synthesizing new elements with $Z > 118$ from the theoretical models appear to be quite large[15–19]. Therefore, it's necessary and very important to examine carefully these three steps in the study of the synthesis mechanism of superheavy nuclei. In this work we only focus on the capture process.

Theoretically, the capture process is often treated as a multi-dimensional barrier penetration problem

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because of the strong couplings between the relative motion and intrinsic degrees of freedom and the coupling to the nucleon transfer channels. On one hand, by solving the coupled-channel (CC) equations to account for these couplings, the CC model has been developed[20–22]. The CC model is very successful in describing fusion excitation functions near the Coulomb barrier[23]. However, for heavy systems, a large number of channels have to be taken into account in the CC model which is not easy to realize. Therefore, full CC calculations become intractable in many cases including many fusion reactions leading to superheavy nuclei. On the other hand, the couplings to other channels split the original single barrier into a set of discrete barriers[22, 24, 25]. These discrete barriers distribute around the original single barrier, each of them has a certain weight representing the probability of encountering the corresponding barrier. Based on the concept of the barrier distribution, a barrier distribution was introduced to take into account the coupled-channel effects in an empirical way, thus several empirical CC approaches have been developed[12, 26–30]. The main differences of these empirical CC approaches are the shape of the barrier distribution and the way of determining the parameters of the barrier distribution. Actually, these empirical CC approaches provide an alternative to the full CC calculations, especially in the cases where the full CC calculations become intractable.

Recently, we have developed an empirical coupled-channel (ECC) model by performing a systematic study of these capture excitation functions[31]. In Ref. [31], the barrier distribution is taken to be an asymmetric Gaussian form and we proposed empirical formulas to calculate the parameters of the barrier distribution. We collected and compiled the measured capture (fusion) excitation functions of 220 reaction systems. The comparison between the calculated cross sections and the experimental ones shows that the ECC model together with these empirical formulas works quite well at energies around the Coulomb barrier and can give a reasonably adequate and systematic description of the capture excitation functions of these 220 reaction systems. In addition, this model has been used to study the CC effects in fusion reactions ^{32}S+^{94,96}Zr and ^{40}Ca+^{94,96}Zr[32] and extended to describe the complete fusion cross sections for the reactions involving weakly bound nuclei at above-barrier energies[33]. Recently, the capture excitation functions of reactions ^{46,50}Ti+^{124}Sn were newly measured and reported[34]. In this work, we will review the ECC model briefly and use this model together with the universal fusion function (UFF) prescription to investigate the dynamical effects in reactions ^{46,50}Ti+^{124}Sn.

The paper is organized as follows. In Sec. 2, we review the ECC model briefly. In Sec. 3, the ECC model and the UFF prescription are applied to analyze the measured capture excitation functions of reactions ^{46,50}Ti+^{124}Sn. Finally, a summary is given in Sec. 4.

2 Empirical coupled-channel model

The capture cross section at a given center-of-mass energy $E_{\text{c.m.}}$ can be written as the sum of the cross section for each partial wave J,

$$\sigma_{\text{capture}}(E_{\text{c.m.}}) = \frac{\pi\hbar^2}{2\mu E_{\text{c.m.}}}\sum_{J=0}^{J_{\max}}(2J+1)T(E_{\text{c.m.}},J), \tag{1}$$

where μ denotes the reduced mass of the reaction system and T denotes the penetration probability. $J_{\max}$ is the critical angular momentum.

In the ECC model[31], a barrier distribution function $f(B)$ is introduced to take into account the coupled-channel effects in an empirical way. Then, the penetration probability is calculated as

$$T(E_{\text{c.m.}},J) = \int f(B)T_{\text{HW}}(E_{\text{c.m.}},J,B)\text{d}B. \tag{2}$$

T_{HW} denotes the penetration probability calculated by the well-known Hill-Wheeler formula[35]. Note that for very light reaction systems and deep sub-barrier penetration, the Hill-Wheeler formula is not valid because of the long tail in the Coulomb potential. In Ref. [36], a new barrier penetration formula was proposed for potential barriers containing a long-range Coulomb interaction and this formula is especially appropriate for the very light reaction systems and the barrier penetration with incident energy much lower than the Coulomb barrier.

The barrier distribution $f(B)$ is taken to be an asymmetric Gaussian function

$$f(B) = \begin{cases} \dfrac{1}{N}\exp\left[-\left(\dfrac{B-B_{\text{m}}}{\varDelta_1}\right)^2\right], & B < B_{\text{m}} \\ \dfrac{1}{N}\exp\left[-\left(\dfrac{B-B_{\text{m}}}{\varDelta_2}\right)^2\right], & B > B_{\text{m}} \end{cases}. \tag{3}$$

$f(B)$ satisfies the normalization condition $\int f(B)\text{d}B = 1$. N is a normalization coefficient. $\varDelta_1$, $\varDelta_2$, and B_{m} denote the left width, the right width, and the central value of the barrier distribution, respectively.

Within the ECC model[31], the barrier distribution is related to the couplings to inelastic excitations (low-lying collective vibrational states and rotational states) and the coupling to the positive Q value neutron transfer (PQNT) channel. The vibrational modes are connected to the change of nuclear shape and nuclear rotational states are related to static deformations of the interacting nuclei. When the two nuclei come close enough to each other, both nuclei are distorted owing to the nuclear and the Coulomb forces, thus dynamical deformations develop[19, 37]. Considering the dynamical deformation, a two-dimensional potential energy surface (PES) with respect to relative distance R and quadrupole deformation of the system can be obtained. Based on the PES, empirical formulas for calculating the parameters of the barrier distribution were proposed to take into account the effect of the couplings to inelastic excitations in Ref. [31]. Note that such empirical formulas are connected with the quadrupole deformation parameters predicted by the finite-range droplet model (FRDM)[38].

The coupling to PQNT channel was suggested to explain the extra enhancement of sub-barrier fusion cross sections of ^{58}Ni+^{64}Ni system as compared with those of ^{58}Ni+^{58}Ni and ^{64}Ni+^{64}Ni systems[39]. Extra enhancements of sub-barrier fusion cross sections have been also observed in most of other reaction systems with PQNT channel. For some of these systems, the fusion excitation functions have been measured in sufficiently small energy steps, which can be used to extract the underlying barrier distributions to study the contribution from transfer channels. The experimental barrier distributions are much broader than those of the reaction systems with negative Q value neutron transfer channel. In this ECC model, the effect of the coupling to the PQNT channel is simulated by broadening the barrier distribution. When the Q value for two-neutron transfer is positive, the widths of the barrier distribution are calculated as $\Delta_i^{'} = gQ(2\text{n}) + \Delta_i, (i=1,2)$, where $Q(2\text{n})$ is the Q value for two-neutron transfer. g is taken as 0.32 for all reactions with positive Q value for two-neutron transfer channel[31].

3　Results and discussions

As mentioned above, the ECC model together with the empirical formulas works quite well at near-barrier energies and can give a reasonably adequate and systematic description of the capture excitation functions of these 220 reaction systems. In addition, for the reactions involving weakly bound nuclei, the systematics of suppression of complete fusion at above-barrier energies have been investigated by using this ECC model[33], the results show that the suppression factors are consistent with those obtained from the universal fusion function (UFF) prescription[40]. Recently, the capture excitation functions of ^{46,50}Ti+^{124}Sn have been measured[34]. In the present work, we use this ECC model to study the CC effects in these two reactions. Furthermore, these two newly measured capture excitation functions can also be used to check the predictive capability of the ECC model.

As we know, the capture excitation function is influenced by two types of features related to the structure of and the interaction potential between the projectile and the target. One is of a static nature, such as the heights, radii, and curvatures of the barriers, and the static effects associated with the excess protons or neutrons in weakly bound nuclei. The other one is the dynamical effects of couplings to inelastic excitations, the breakup channel, and nucleon transfer channels. In order to study the dynamical coupling effects on capture cross sections directly, it is necessary to eliminate the geometrical factors and static effects of the potential between the two nuclei[41, 42].

For reactions ^{46,50}Ti+^{124}Sn, we first investigate the dynamical coupling effects on capture cross sections by eliminating the static effects using the UFF prescription[41, 42]. According to this prescription, the capture cross section and the collision energy are reduced to a dimensionless fusion function $F(x) = 2E_{\text{c.m.}}\sigma_{\text{capture}}/R_{\text{B}}^2\hbar\omega$ and a dimensionless variable $x = (E_{\text{c.m.}} - V_{\text{B}})/\hbar\omega$. Here $E_{\text{c.m.}}$ is the collision energy in the center-of-mass frame, σ_{capture} is the capture cross section, and V_{B}, $\hbar\omega$, and R_{B} denote the height, curvature, and radius of the barrier which is approximated by a parabola. The barrier parameters V_{B}, $\hbar\omega$, and R_{B} are obtained from the double folding and parameter-free São Paulo potential (SPP)[43–45].

The reduced capture excitation functions of the reactions ^{46}Ti+^{124}Sn and ^{50}Ti+^{124}Sn are shown in Fig. 1. The solid line represents the UFF. One can see that the reduced capture cross sections for ^{46}Ti+^{124}Sn and ^{50}Ti+^{124}Sn are both larger than the UFF at sub-barrier energies. Furthermore, the sub-barrier capture cross sections of ^{46}Ti+^{124}Sn show an extra enhancement as compared with those of ^{50}Ti+^{124}Sn. This implies that the enhancement of the sub-barrier capture cross sections due to the coupling effects in ^{46}Ti+^{124}Sn is much larger than that in ^{50}Ti+^{124}Sn. The reason is that the Q value of two neutrons transfer channel is positive for ^{46}Ti+^{124}Sn but negative for ^{50}Ti+^{124}Sn. The $Q(2n)$s are 6.072 MeV and −0.255 MeV for these two reaction systems, respectively.

Next, we adopt the ECC model to investigate the effects of the couplings to inelastic excitations and neutron transfer channels. The comparison between the calculated and measured capture excitation functions for ^{46}Ti+^{124}Sn and ^{50}Ti+^{124}Sn is shown in Fig. 2. For ^{46}Ti+^{124}Sn, the calculated parameters Δ_1, Δ_2, and

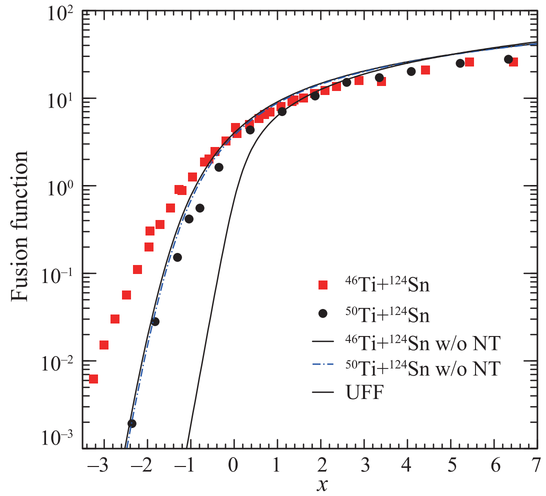

Fig. 1 (color online) The reduced capture function for reactions ^{46}Ti+^{124}Sn and ^{50}Ti+^{124}Sn as a function of x. The solid line represents the UFF. The data are taken from Ref. [34]. The dotted (dash-dotted) line shows the reduced fusion function calculated for ^{46}Ti+^{124}Sn (^{50}Ti+^{124}Sn) with neutron transfer (NT) effect not included.

$B_{\rm m}$ are 4.17 MeV, 8.41 MeV, and 124.25 MeV, respectively, while for ^{50}Ti+^{124}Sn, the calculated parameters $\varDelta_1$, $\varDelta_2$, and $B_{\rm m}$ are 2.15 MeV, 6.25 MeV, and 123.01 MeV, respectively, The dotted line denotes the results with only considering the couplings to inelastic excitations. One can find that the dotted line is in good agreement with the data for ^{50}Ti+^{124}Sn, while the theoretical results underestimate the sub-barrier cross sections considerably for ^{46}Ti+^{124}Sn. This means that the couplings to inelastic excitations are responsible for the enhancement of the sub-barrier cross sections for ^{50}Ti+^{124}Sn. Moreover, the results with only considering the couplings to inelastic excitations are reduced by using the UFF prescription, which are shown in Fig. 1 by the dotted and dash-dotted lines. One can see that the dotted line coincides with the dash-dotted line. It implies that the effects of the couplings to inelastic excitations are almost the same in these two reactions.

As discussed above, the extra enhancement of the sub-barrier cross sections for ^{46}Ti+^{124}Sn is ascribed to the effect of the couplings to PQNT channels. Then the results with the neutron transfer effect taken into account are shown in Fig. 2 by the solid line. One can see that, after considering the neutron transfer effect, the theoretical results are in good agreement with the data for ^{46}Ti+^{124}Sn. These results show that the ECC model reproduces the data of these two reactions quite well. Note that there is no free parameters in these calculations. This implies that the coupling to PQNT channel is very important and necessary to explain the experimental data and the positive Q value of two neutrons transfer is very important to understand and describe the PQNT effect. Up to now, the role of the PQNT effect is still not very clear. Further experimental and theoretical studied are needed.

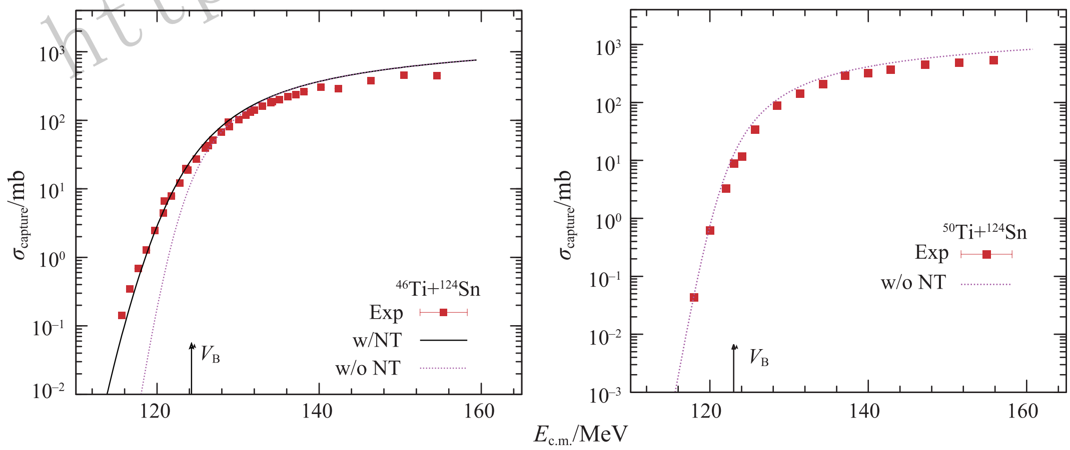

Fig. 2 (color online) The capture excitation functions for ^{46}Ti+^{124}Sn and ^{50}Ti+^{124}Sn. The solid line denotes the calculated cross sections with the NT effect taken into account. The dotted lines denote the calculated cross sections with the NT effect neglected. The arrow indicates the central value of the barrier distribution. The data are taken from Ref. [34].

4 Summary

The ECC model combined with the universal fusion function prescription is adopted to study the coupled-channel effects in fusion reactions ^{46,50}Ti+^{124}Sn which were recently measured. The reduced fusion functions show that the sub-barrier capture cross sections of ^{46}Ti+^{124}Sn exhibit an extra enhancement as compared with those of ^{50}Ti+^{124}Sn. The results from the ECC model are in good agreement with the data and show that the extra enhancement of the sub-barrier cross sections for ^{46}Ti+^{124}Sn can be ascribed to the positive Q value neutron transfer effect. Therefore, the predictive capability of the ECC model has been checked to some extent by the reproduction of the two newly measured capture excitation functions. We expect that this ECC model can provide proper predictions of capture cross sections for the synthesis of superheavy nuclei.

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^{46,50}Ti+^{124}Sn 熔合反应中耦合道效应的理论研究

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摘要： 近库仑位垒重离子俘获与熔合是一个典型的多位垒穿透过程。在本征道的理论框架下，多反应道的耦合会使得单个位垒分离成一系列的分立位垒。基于位垒分布的思想，我们最近发展了一个经验的耦合道 (ECC) 模型，并系统地研究了 220 个反应体系的俘获激发函数。最近，实验报道了熔合反应 ^{46,50}Ti+^{124}Sn 俘获激发函数的测量结果。本文将简要介绍该 ECC 模型，并结合通用熔合函数 (UFF) 的约化方法，利用该模型研究熔合反应 ^{46,50}Ti+^{124}Sn 中的耦合道效应。UFF 的约化结果表明，相比于 ^{50}Ti+^{124}Sn，^{46}Ti+^{124}Sn 的垒下俘获截面有额外的增强。ECC 模型成功地再现了实验测得的俘获激发函数，并表明，^{46}Ti+^{124}Sn 垒下俘获截面的额外增强来源于正 Q 值的中子转移效应。

关键词： 位垒分布；经验耦合道模型；耦合道效应

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