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A Universal Formula for Light Attenuation of Scintillator Detector

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Abstract: Scintillator detectors are widely used in modern nuclear and particle physics experiments. Studying the light attenuation of scintillator detector (LASD) is vitally important for extracting proper measurements of energy and time. In this paper, we integrate the isotropic fluorescence over solid angle to study the influence on overall light-intensity from varying optical path at different angle. Based on numerical results, a universal formula for describing LASD is derived. Under certain condition, our formula can be written as a form of widely-used double-exponential function. The universal formula describes the experimental data of PSD at DAMPE, reducing the maximum deviation at far-side of the scintillator from ∼10% to less than 2%. Moreover, our model also deciphers Kaiser's experiment, Gierlik's experiment and Platino's experiment successfully.

Key words: scintillator detector; light attenuation

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1 Introduction

Scintillator detectors are widely used as time-of-flight detector and electromagnetic calorimeters, such as CMS at CERN[1], BESIII at IHEP[2], PSD at DAMPE[3–6] and so on. When charged particles enter the scintillator, they would deposit their energy by ionization, and fluorescence lights will be generated along their pathes. The emitted fluorescence lights propagate to ends of the scintillator and are collected by photomultiplier tubes (PMT) typically. Studying the light attenuation behavior of scintillator detector, combining with the hit position and the signal output, is a very important step to obtain the deposited energy of charged particle. Light attenuation of bulk scintillator can be described by an exponential function. However, the light attenuation behaviors of scintillator detector also depend on the properties of detector itself like geometrical shape, wrapping material, surface roughness and so on. The double exponent (DE) model[7] and the reflected back (RB) model[8] were two widely used empirical formulas to describe light attenuation behavior of strip scintillator detectors. We found that DE and RB models can not perfectly describe the light attenuation behaviors of DAMPE PSD scintillator detector (see Sec. 2), therefore a more universal model to describe the LASD is derived in this work.

The paper is organized as follows. A brief introduction to the exponential decay (ED) model, the DE model and the RB model is given in Sec. 2. Sec. 3 describes our model for fluorescence light propagation in a scintillator and points out that a non-constant modification fraction is necessary for considering the fluorescence light to be equivalent parallel light. Sec. 4 shows the fitting result of our formula. Sec. 5 illustrates the universality of our model. Finally, conclusions are given in Sec. 6.

2 Existing formulas for describing the light attenuation of scintillator detector

Stripe-like scintillator detector is widely used in high energy experiment. When charged particles hit

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the scintillator, some energy is deposited in the scintillator material. As a consequence, atoms and molecules in the scintillator are stimulated to an excited state. When they return back to the ground state, they will emit fluorescence light. The emitted lights are collected by the PMT at the end of scintillator typically. Thus, the information about the impinging particles can be extracted by analyzing the electronic signals from the PMT. It is generally known that the attenuation of transmitted light with an initial intensity I_0 in a bulk medium can be described by an ED function,

$$I(x)=I_0\mathrm{e}^{-x/\lambda}, \tag{1}$$

where x is the position along the scintillator detector and the parameter λ is the so-called attenuation length.

In the past several decades, two further physical formulas were proposed: the DE formula and the RB formula.

Kaiser *et al.*[7] proposed the DE formula of light transmission:

$$I(x)=I_1\mathrm{e}^{-x/\lambda_1}+I_2\mathrm{e}^{-x/\lambda_2}, \tag{2}$$

where λ_1 denotes a short attenuation length with the order of a few centimeters, and λ_2, ranging from 1 to several metres, denotes long attenuation length. I_1 and I_2 are constant terms for λ_1 and λ_2 respectively.

Besides, Taiuti *et al.*[8] proposed the RB formula where a fraction η of the emitted light could be reflected back from the other end, which is added to the initial light as another component. This formula can be described by the expression:

$$I(x)=I_0(\mathrm{e}^{-x/\lambda}+\eta\mathrm{e}^{-(2L-x)/\lambda}). \tag{3}$$

We found that neither DE nor RB models can perfectly describe the light attenuation behaviors of DAMPE PSD scintillator detector. Fig. 1(a) and (c) show the light attenuation data of a DAMPE PSD scintillator bar (data points are taken from Ref. [5]), while the fitted DE and RB formulas are shown by red solid lines, respectively. The deviations between the fitted functions and experimental data are calculated bin-by-bin,

$$\delta(x)=\frac{I_{\mathrm{fit}}(x)-I_{\mathrm{exp}}(x)}{I_{\mathrm{exp}}(x)}. \tag{4}$$

In Fig. 1(b) and (d), the deviations are about 2%~4% in the most of hit position regions, but devia-

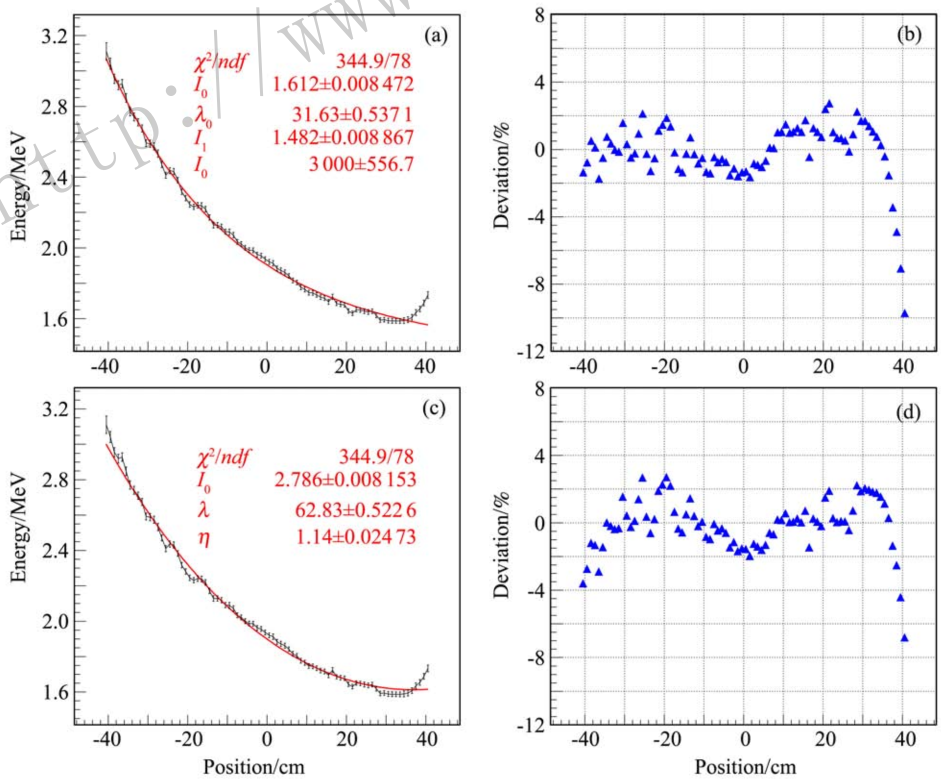

Fig. 1 (color online) (a) The curve fitted with the samples from PSD at DAMPE[4–5] by the Double Exponent formula after converting length coordinates to position coordinates. (b) The deviation between the Double Exponent formula fitting results and the samples. (c) The curve fitted with the same samples by the Reflected Back formula. (d) The deviation between the Reflected Back formula fitting results and the samples.

tions in the far-side of the detector get worse rapidly. This implies that the ED and RB formula can describe the most part of the measured light attenuation of PSD on DAMPE[4–5], but both of them are failed to describe the data at far-side of the scintillator. Therefore we search for precise physical formulas to fit this data samples.

3 A universal formula for light transport in scintillator

Attenuation of parallel light in scintillator follows an exponential law. The fluorescence is emitted isotropically and the optical length depends on polar angle and axial distance x, we need to sum up contributions from different solid angles to get the overall intensity at a certain axial distance.

For simplicity, a right circular scintillator detector with a radius $r_{\rm base}$ and a height L is considered. The collection PMT is a right circular cylinders with a radius $r_{\rm PMT}$, which is connected by a right conical frustum to the scintillator edge. The base angle of connector is α, and the position of emitted light on the axis is denoted as x, while the direction of an emitted photon with respect to the cylinder axis is θ, as shown in Fig. 2.

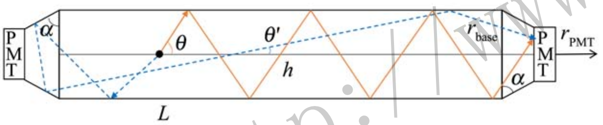

Fig. 2 (color online) The schematic diagram of scintillator. It's a right circular cylinders with base radius $r_{\rm base}$ and length L and the PMT is a right circular cylinders with base radius $r_{\rm PMT}$ connected with the scintillator edge by a right conical frustum (light guide), whose base angle is α. The distance of emitted light on the axis is x, θ is the direction of an emitted photon with respect to the cylinder axis, and θ' is the axial angle of reflected light.

Each photon emitted at angle θ has an optical path length $x/\cos\theta$ before it enters the light gride, and the emitted light is 4π uniformly distributed. Signal collected from $\theta\in[0,\pi/2]$ and $\theta\in[\pi/2,\pi]$ are respectively labelled as I_a and I_b.

$$\begin{aligned} I_a(x) &= I_0\frac{2\pi}{4\pi}\int_0^{\pi/2} \mathrm{e}^{-\frac{x}{\lambda\cos\theta}}\xi^{n(x,\theta)}R_a\sin\theta\mathrm{d}\theta \\ &= I_0\eta_a(x)\mathrm{e}^{-\frac{x}{\lambda}}, \end{aligned} \tag{5}$$

where λ is the attenuation length of bulk scintillator, ξ is the reflection factor, R_a means the effective collection fraction for photons at the end of scintillator and the n corresponds to the number of times for reflections. Here, we can get n as a function of x and θ.

$$\begin{aligned} &\left\lfloor \frac{x}{r_{\rm base}\cot\theta}\right\rfloor : 1,2,3,4,5,\ldots \to n: 1,1,2,2,3,\ldots, \\ &n(x,\theta) = \left\lfloor \frac{\left\lfloor\frac{x}{r_{\rm base}\cot\theta}\right\rfloor+1}{2}\right\rfloor = \left\lfloor \frac{x/r_{\rm base}+\cot\theta}{2\cot\theta}\right\rfloor, \end{aligned} \tag{6}$$

where $\lfloor x\rfloor$ is the floor function. Then,

$$\eta_a(x) = \frac{2\pi}{4\pi}\int_0^{\pi/2}\mathrm{e}^{\frac{x}{\lambda}}\mathrm{e}^{-\frac{x}{\lambda\cos\theta}}\xi^{n(x,\theta)}R_a\sin\theta\mathrm{d}\theta. \tag{7}$$

Similarly,

$$\begin{aligned} I_b(x) =& I_0\frac{2\pi}{4\pi}\int_{\pi/2}^{\pi}\mathrm{e}^{-\frac{(L-x)}{\lambda|\cos\theta|}}\xi^{n(L-x,\theta)}(1-R_a)\times \\ & \mathrm{e}^{\frac{-L}{\lambda|\cos(\theta')|}}\xi^{n(L,\theta')}R_a\sin\theta d\theta \\ =& I_0\eta_b(x)\mathrm{e}^{-\frac{(2L-x)}{\lambda}}, \end{aligned} \tag{8}$$

where θ' is the axial angle after reflecting from another end. We consider two main situations, in which lights are reflected by one time or two times.

$$\theta' = \begin{cases} 3\theta-2\alpha, & \text{if } \pi/2<\theta<\pi-\alpha, \text{ (single)} \\ \theta+4\alpha, & \text{if } \pi-\alpha<\theta<3\pi/2-2\alpha, \text{ (double)} \end{cases} \tag{9}$$

then,

$$\begin{aligned} \eta_b(x) =& \frac{2\pi}{4\pi}\int_{\pi/2}^{\pi}\mathrm{e}^{\frac{(2L-x)}{\lambda}}\mathrm{e}^{-\frac{(L-x)}{\lambda|\cos\theta|}}\xi^{n(L-x,\theta)}(1-R_a)\times \\ & \mathrm{e}^{\frac{-L}{\lambda|\cos(\theta')|}}\xi^{n(L,\theta')}R_a\sin\theta\mathrm{d}\theta. \end{aligned} \tag{10}$$

As the analytical solution of $\eta_a(x)$ and $\eta_b(x)$ are inaccessible, we can firstly get the numerical solutions and then fit them with empirical formulas. The numerical solutions are shown in Fig. 3, we find that the exponential formula and inverse proportional formula, *i.e.*,

$$\eta_a(x) = a_0\mathrm{e}^{\frac{a_1}{a_2+x}},\quad \eta_b(x) = \frac{a_3}{1-x/a_4},$$

are appropriate approximations for different parameter sets. This means that we can use this empirical formula to simplify the following calculations. Then, we have

$$I(x) = I_0\left(\eta_a(x)\mathrm{e}^{-x/\lambda}+\eta_b(x)\mathrm{e}^{-(2L-x)/\lambda}\right). \tag{11}$$

where parameter a_0 can be absorbed into I_0 and a_3, and we have

$$\begin{aligned} \eta_a(x) &= \exp\left(\frac{a_1}{a_2+x}\right), \\ \eta_b(x) &= \frac{a_3}{1-x/a_4}. \end{aligned} \tag{12}$$

This formula describes a diffused light source as an equivalent parallel source with modification fractions $\eta_a(x)$ and $\eta_b(x)$. Note that the parameter a_3 decreases as the R_b decreases, meaning the contribution from

$\theta \in [\pi/2, \pi]$ decreases as well. Especially, when $R_b \to 0$, the a_3 tends to zero, which means there will be no reflection contribution. The increase of reflection factor ξ will cause both parameters a_1 and a_3 increase accordingly, this corresponds to the reduction of lost from $\theta \in [0, \pi/2]$ and the increase of contribution from $\theta \in [\pi/2, \pi]$. Parameters a_2 and a_4 increase as attenuation length λ, besides, they eliminate the singular points of the coefficient at x=0 and x=L.

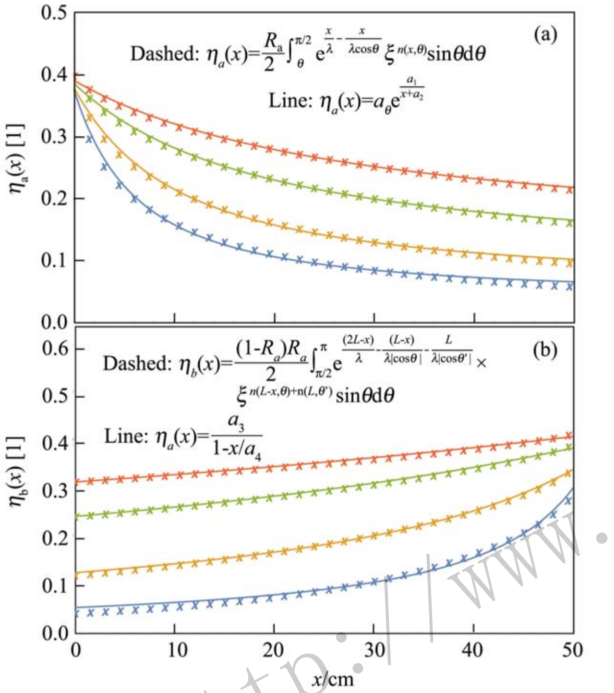

Fig. 3 (color online) The numerical results (x-dashed line) and empirical formulas (thick line) of η_a (a) and η_b (b). Curves with different colors correspond to different parameter sets, L=50 cm, ξ=1, $\alpha = \pi/3$, R_a=0.8, λ=100, 50, 20, 10 cm from up to down. The exponential empirical formulas show a good fitting result and fitting parameters are shown in Table 1.

Table 1 The fitting parameter in Fig. 3.

λ/cm	a_0	a_1	a_2	a_3	a_4
100	0.036	39.7	17.0	0.055	60.9
50	0.053	48.3	24.6	0.129	79.6
20	0.088	54.9	37.1	0.248	135.6
10	0.124	55.4	48.2	0.320	216.9

4 Results

4.1 Fitting the same samples from PSD at DAMPE

Fitting the same samples from PSD at DAMPE with our formula after converting length coordinates to position coordinates, the maximum deviation at far-side of the scintillator is reduced to less than 2%. Fig. 4 and Table. 2 show that this is better than the DE formula (10%) and the RB formula (7%).

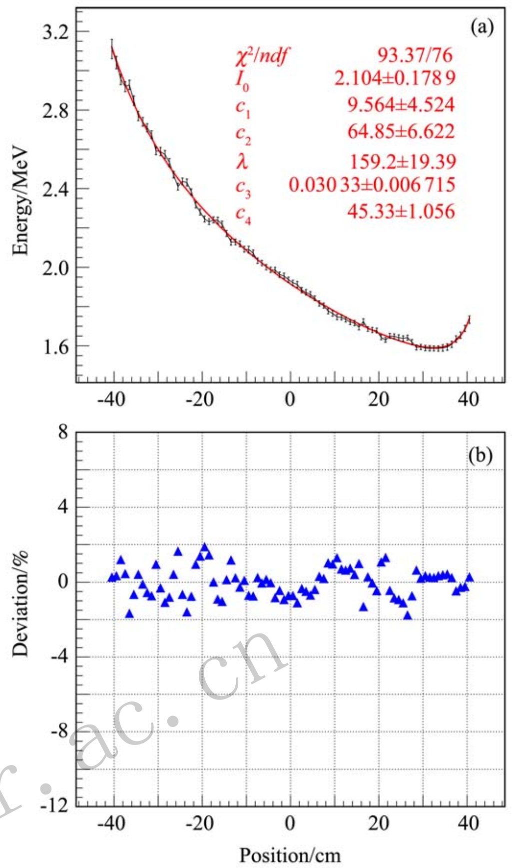

Fig. 4 (color online) (a) The curve fitted with the samples from PSD at DAMPE by our model after converting length coordinates to position coordinates. (b) The deviation between our new formula fitting results and the samples.

Table 2 Compare the deviation with other formulas at far-side of the scintillator.

Item	DE formula	RB formula	Our formula
$\delta_{\max}$	10%	7%	<2%
χ^2/ndf	344.9/78	278.1/79	93.4/76

4.2 More experimental data fitting

To have a better insight into our formula, we have considered various experimental data from different sources.

Kaiser *et al.*[7] performed a study of the light emitted from commercial plastic scintillators using photomultiplier tubes with S-11 and S-13 responses, giving their light yield *vs.* length. The left one of Fig. 5(a) shows that our formula is consistent with their data.

Taiuti *et al.*[8] investigated the behaviour of plastic scintillator bars with length up to 450 cm, which is designed for the CLAS large angle electromagnetic shower calorimeter, and gave their light output with different lengthes and different materials. The right one of Fig. 5(b) shows that our formula is also consistent with their data.

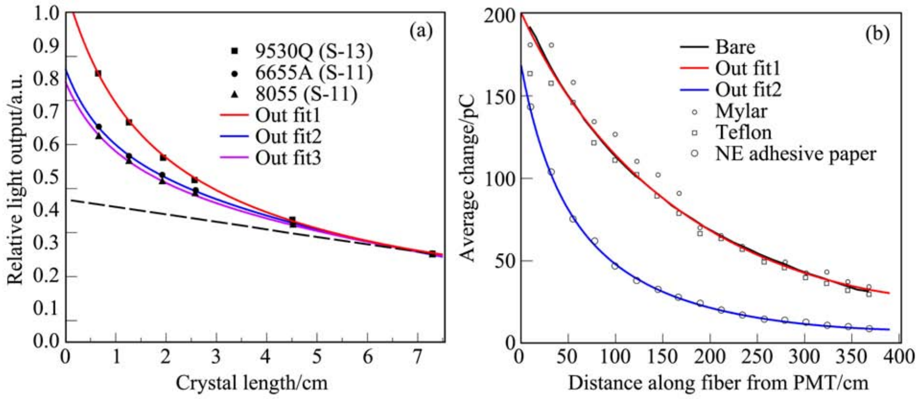

Fig. 5 (color online) (a) All the data are taken from Kaiser′s experiment about light output *vs.* length[7], and the three solid lines are our formula fitting results. (b) All the data are taken from Taiuti′s experiment about light output with length[8], while the two solid lines are our formula fitting results.

Gierlik *et al.*[9] investigated the light transport properties of 200 mm×6 mm×6 mm BC400 plastic bars and of other samples of BC408, giving their light output *vs.* lengthes with different environments and different plastic bars. Fig. 6(a) and (b) show that our formula is also consistent with their data.

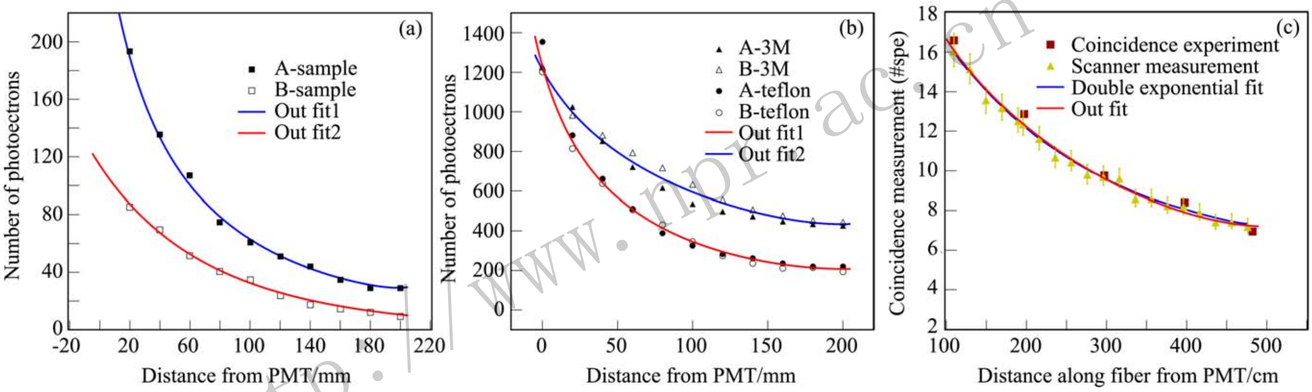

Fig. 6 (color online) (a) and (b) All the data are taken from Gierlik′s experiment about light output *vs.* length[9], the solid lines are our formula fitting results, (c) the data are taken from Platino′s experiment about light output with length[10], the blue line is the double exponent (DE) formula fitting result, while the red line is our formula fitting result.

Platino *et al.*[10] presented the fabrication, testing and initial calibration system of scintillator modules to be used as muon counters for cosmic ray particle shower detection, giving their light output *vs.* length in Fig. 6(c). Still, our formula shows consistency with their data.

5 Discussion

Exactly, our formula is derived in an approximation situation for cylindrical scintillator, rather than for the square strip in reality. The fitting results show what really matters is the contribution of θ-integration. It is interesting that our new model can be reduced to the three existing formulas from Sec. 2 under different circumstances.

Case 1: for $a_1=0$ and $a_3=0$, our formula can be equivalent to the ED formula.

Case 2: for $a_1=0$ and $a_4 \gg L$, our formula can be equivalent to the RB formula.

Case 3: for $a_3=0$, λ is about metres and larger than L, our formula can be approximately reduced to the DE formula (see Appendix for details).

It is worth mentioning that the reflections of light at the ends of the scintillator was developed by Chipaux *et al.*[11] by a semi-empirical way. Our formula can also be approximately related to that semi empirical formula, in the case of $a_3=0$. Therefore, our formula is universal as it's applicable for all above cases.

6 Conclusions

Our formula for light transport in scintillators consider the isotropic fluorescence as an effective parallel light by including modification fractions that comes from the integration of space angle. It can fit the data from DAMPE very well, and reduce the deviation between fitting function and the data samples from 10% to less than 2%. In addition, this developed formula can fit Kaiser's experiment, Gierlik's experiment and Platino's experiment successfully.

Acknowledgments: We thank E. Ciuffoli for many insightful discussions.

Appendix:

For case 3, a_3=0, then $I(x)=I_0\mathrm{e}^{\frac{a_1}{a_2+x}}\mathrm{e}^{-x/\lambda}+I_0\frac{a_3}{1-x/a_4}\mathrm{e}^{-(2L-x)/\lambda}=I_0\mathrm{e}^{\frac{a_1}{a_2+x}}\mathrm{e}^{-x/\lambda}+I_0\frac{0}{1-x/a_4}\mathrm{e}^{-(2L-x)/\lambda}=I_0\mathrm{e}^{\frac{a_1}{a_2+x}}\mathrm{e}^{-x/\lambda}$ and λ is in the order of metres. For the DE formula, with λ_1 being in the order of a few centimetres and λ_2 ranging from 1 to several metres, $I_{\mathrm{DE}}(x)=I_1\mathrm{e}^{-x/\lambda_1}+I_2\mathrm{e}^{-x/\lambda_2}=I_2\mathrm{e}^{-x/\lambda_2}\left(1+\frac{I_1}{I_2}\mathrm{e}^{-x/\lambda_1}\mathrm{e}^{x/\lambda_2}\right)=I_2\mathrm{e}^{-x/\lambda_2}\left(1+\frac{I_1}{I_2}\mathrm{e}^{\frac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}x}\right)=\left(I_2+I_1\mathrm{e}^{-x/\lambda_1}\right)\mathrm{e}^{-x/\lambda_2}$.

Choosing λ to be λ_2 and $I_2+I_1\mathrm{e}^{-x/\lambda_1}$ to be close to $I_0\mathrm{e}^{\frac{a_1}{a_2+x}}$ means our formula will be equivalent to DE formula. In Fig. 7 shows that our formula is close to DE formula with different parameter sets.

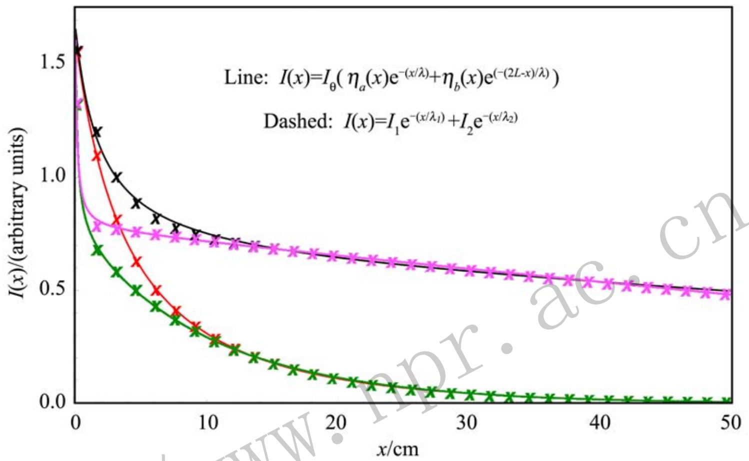

Fig. 7 (color online) The DE formula (dashed line) and our formula (thick line). Curves with different colors correspond to different parameter sets, $I_1=I_2=0.8$, $\{\lambda_1, \lambda_2\}$ are $\{2.5,10\}$, $\{2.5,100\}$, $\{0.25,10\}$, $\{0.25,100\}$ [cm] for red, black, green, yellow-dashed line respectively. They are almost the same.

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闪烁体探测器中光的衰减通式

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摘要： 闪烁体探测器被广泛应用于当今粒子物理与原子核物理实验中。研究闪烁探测器的光衰减规律(LASD)对时间和能量的准确测量都十分重要，这一点对条形闪烁探测器尤为如此。本文以圆柱闪烁探测器为例，对各向同性的闪烁光进行立体角积分，进而研究不同立体角下光程差异对结果的影响。在数值计算的基础上，导出了描述LASD的通用公式。在一定条件下，公式可以约化为双指数衰减形式。对于DAMPE上PSD的实验数据，该公式能使闪烁体远端的拟合偏差从大约10%降低至2%以下。同时，模型也能够很好地描述Kaiser实验、Gierlik实验和Platino实验的实验数据。

关键词： 闪烁探测器；光衰减

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